

## **Elements of the Instructional Families: Number and Operations – Fractions**

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## **Teaching Fractions**

All the CCCs in this document relate to teaching Fractions. Below are some additional resources that may be helpful:

[Teaching Ideas](#)

[Math Lessons](#)

[Math Games](#)

[Math Lessons and Tutorials](#)

[Math Test Prep](#)

[Math Journal Articles](#)

## CCC Mathematics | Number Operations (Fractions)

**CCSS:** 3.NF.1 Understand a fraction  $\frac{1}{b}$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $\frac{a}{b}$  as the quantity formed by  $a$  parts of size  $\frac{1}{b}$ .

**CCC:** 3.NO.111 Identify the number of highlighted parts (numerator) of a given representation (rectangles and circles).

**Strand:** Number Operations (Fractions, Ratios, Proportions)

**Family:** Representing

**Progress Indicator:** *E.NO.11 Identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers*

### Concrete Understandings:

- Understand that fractions are equal parts of a whole (i.e., Describe that the denominator of a fraction represents the number of equal parts within a whole (length unit or region).
- Count using one-to-one correspondence

### Representation:

- Apply understanding that the numerator represents the parts of the whole (i.e., how many of the parts are being considered). Begin with unit fraction which are fractions with a numerator of 1.
- Understand the following concepts, symbols, and vocabulary: numerator, fraction, equal parts.

### Suggested Instructional Strategies:

- Folding a sentence strip into 2, 4, and 8 equal pieces
- Folding a sentence strip into 3 and 6 equal pieces
- Model-Lead-Test\*
- Partitioning: Breaking an object or set of objects into pieces
- Pizza Fractions: Using cutout of pizza/pizza circle with fractions written on them that can be placed on a fraction template
- Ruler and/or yardstick: Use a ruler or yardstick with sticky dots on it to indicate whole or parts on a number line

### Supports and Scaffolds:

- Visual models with pre-marked and pre-divided regions
- Graph paper
- Manipulatives
- Rectangles and circles with raised edges on highlighted section
- Assistive technology
- iPad applications

- Objects (e.g., apples) shared equally and matched with a fraction card

**CCSS:** 3.NF.1 Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .

**CCC:** 3.NO.112 Identify the total number of parts (denominator) of a given representation (rectangles and circles).

**Progress Indicator:** *E.NO.11 Identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers*

**Concrete Understandings:**

- Understands that fractions are equal parts of a whole (i.e., Describe that the denominator of a fraction represents the number of equal parts within a whole (length unit or region).
- Count using one-to-one correspondence.

**Representation:**

- Understand that the denominator of the fraction indicates the number of equal parts of the parts.
- Understand the following concepts, symbols, and vocabulary: denominator, fraction, equal parts.

**Suggested Instructional Strategies:**

- Folding a sentence strip into 2, 4, and 8 equal pieces
- Folding a sentence strip into 3 and 6 equal pieces
- Model-Lead-Test\*
- Partitioning: Breaking an object or set of objects into pieces
- Pizza Fractions: Using cutout of pizza/pizza circle with fractions written on them that can be placed on a fraction template
- Ruler and/or yardstick: Use a ruler or yardstick with sticky dots on it to indicate whole or parts on a number line

**Supports and Scaffolds:**

- Visual models with pre-marked and pre-divided regions
- Graph paper
- Manipulatives
- Rectangles and circles with raised edges on highlighted section
- Assistive technology
- iPad applications
- Objects (e.g., apples) shared equally and matched with a fraction card

**CCSS:** 3.NF.1 Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by a parts of size  $1/b$ .

**CCC:** 3.NO.1I3 Identify the fraction that matches the representation (rectangles and circles; halves, fourths, and thirds, eighths).

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Representing

**Progress Indicator:** *E.NO.1I Identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers*

**Concrete Understandings:**

- Identify the parts of a region and the whole region when a region is partitioned when item is divided.
- Count the number of the parts selected (e.g., 3 of the 4 parts; have fraction present but not required to read  $\frac{3}{4}$ ).

**Representation:**

- Understand how parts of a whole can be expressed as fractions using numbers.
- Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts.
- Recognize that fraction bars of equal lengths can be divided into different numbers of equal parts/units.
- Understand a fraction  $a/b$  as the quantity formed by a parts of size  $1/b$ .
- Ability to recognize that the more equal parts, the smaller the part.
- Understand the following concepts, symbols, and vocabulary: numerator, denominator,  $\frac{\_}{\_}$ .

**Suggested Instructional Strategies:**

- Before introducing fraction, use fraction bars:
  - Describe a fraction bar in multiple ways (e.g., present a fraction bar with four parts and two parts shaded red and describe the representation as the color and the number of parts shaded (a red bar with two parts shaded); four parts and two parts shaded (without using color); or two out of four parts are shaded).
  - Explicitly teach types of fraction bars (whole bars with all parts shaded; whole bars divided into parts with no parts shaded; whole bars with half of the parts shaded).
  - Explicitly teach that parts out of total parts shaded (e.g., two out of four parts shaded) is the language we use to name the fraction ( $\frac{2}{4}$ ).
- Teach fractions explicitly to indicate part of a whole.
- Teach explicitly that as the numerator increases, there will be more parts.
- Multiple Exemplars (e.g., fraction bars and fractions)

- Exemplar/Non-exemplar: Here is picture/representation of  $\frac{1}{2}$  (present a fraction bar). This is the fraction  $\frac{1}{2}$ . This is the fraction  $\frac{1}{2}$ . This is not the fraction  $\frac{1}{2}$ . Show me a fraction bar that represents  $\frac{1}{2}$ .
- Task Analysis
  - Present a shaded fraction bar with the associated fraction.
  - State that the number is called a fraction.
  - State how to determine the fraction (e.g., I have a blue bar with 4 parts/units. Two of the parts are shaded. The fraction is two over four or two-fourths.
  - State that for the fraction (e.g.,  $\frac{2}{4}$ ), the denominator means to divide something into “four” equal parts and the numerator “2” indicates 2 of these parts.
  - Present the fraction and have the student create/select the associated representation of the fraction.
- Have the student give fraction statements that are true for a provided group of objects. For example, 2 out of 3 or  $\frac{2}{3}$  of the pencils are yellow. Show the corresponding fraction.
- Provide “hands on” opportunities to create fractions (e.g., salt dough, pies)

**Suggested Supports and Scaffolds:**

- Geoboards
- Dot-paper
- Cuisenaire rods
- Color tiles
- Pattern blocks or sets of objects
- Pie diagrams
- Fraction bars that are ruled into certain fixed partitions
- Assistive technology
- iPad applications
- Objects (e.g., apples) shared equally and matched with a fraction card

**CCSS:** 3.NF.2a and 2b Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- a) Represent a fraction  $1/b$  on a number line diagram by defined the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line.
- b) Represent a fraction  $a/b$  on a number line diagram by marking off a lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number line.

**CCC:** 3.NO.114 Identify that a part of a rectangle can be represented as a fraction that has a value between 0 and 1.

**Strand:** Number Operations (Fractions, Ratios, Proportions)

**Family:** Representing

**Progress Indicator:** *E.NO.11 Identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers*

**Concrete Understandings:**

Understands that fractions are equal parts of a whole (i.e., Describe that the denominator of a fraction represents the number of equal parts within a whole (length unit or region).

**Representation:**

- Partition the rectangle based on the number in the denominator.
- Understand the following concepts, symbols, and vocabulary: partition, divide, equal parts, fraction.

**Suggested Instructional Strategies:**

- Use multiple exemplar or time delay to teach parts of a whole\*
- Have students show a subset of a set (1 of the 6 objects are red/square/rough)
- Use Model-Lead-Test to demonstrate and teach students to fold sentence strips\*
- Use Model-Lead-Test to demonstrate and teach students draw regions and partition on graph paper\*

**Supports and Scaffolds:**

- Color tiles
- Pattern blocks or sets of objects
- Pie diagrams
- Fraction bars that are ruled into certain fixed partitions
- Assistive technology
- iPad applications
- Objects (e.g., apples) shared equally and matched with a fraction card



**CCSS:** 3.NF.1 Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .

**CCC:** 4.NO.1n1 Select a model of given fraction (halves, thirds, fourths, sixths, eighths).

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Representing

**Progress Indicator:** *E.NO.1n Comparing and modeling fractions, including with different denominators*

**Concrete Understandings:**

- Differentiate between parts of a whole and the whole itself.
- Understand the structure of a fraction (i.e., Describe that the denominator of a fraction represents the number of equal parts within a whole (length unit or region).

**Representation:**

- Understand the components of a fraction (numerator and denominator).
- Understand the following concepts, symbols, and vocabulary: fraction, numerator and denominator

**Instructional Strategies:**

- Teach numerator = part, and denominator = whole using a model. Types of models may include area or region models (e.g., pattern blocks, pie pieces, and grid or dot paper), length models (e.g., number lines, Cuisenaire rods, fraction rods, line segment drawings, etc.), and set models (e.g., drawings using X's and O's, two-color counters in loops on paper).
- Time Delay\*
- Have students demonstrate a fraction by shading in the correct number of units given a fraction bar with 2, 3, 4, 6 or 8, units
- Have students show a subset of a set (1 of the 6 objects are red/square/rough)
- Use multiple exemplar training\*
- Use Least-to-Most prompting\*
- Have the student give fraction statements that are true for a provided group of objects. For example, say, "2/6 of the pencils are yellow. Show the corresponding/matching fraction." Using a shaded fraction bar, say "This show 2 parts out of 6/6 parts with 2 shaded/2 parts shaded out of 6. Show the corresponding/matching fraction."

**Supports and Scaffolds:**

- 2-dimensional rectangle segmented into parts (vs. a pizza)
- Objects to model fractions
- One-dimensional linear models (fraction bars, number lines)
- Software such as Smart boards to create and manipulate models of fractions

**CCSS:** 5.NBT.3a Read, write, and compare decimals to thousandths. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,  $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ .

**CCC:** 5.NO.1b1 Read, write, or select a decimal to the hundredths place.

**Strand:** Number Operations (Fractions/Ratios/Proportions)

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**Progress Indicator:** *M.NO.1b Extending place value understanding to reading (e.g., naming the values with number words, rather than “point four”), writing, comparing, and rounding decimals*

**Concrete Understandings:**

- Recognize part/whole when materials are divided into tenths.
- Count tenths to determine how many (e.g., 4 tenths: 0.4). Have the decimal present but student is not required to read.

**Representation:**

- Count to 100.
- Understand place value to the hundredths.
- Understand where to write a decimal point.
- Understand concepts, symbols, and vocabulary: decimal, decimal point, tenths place, hundredths place.

**Suggested Instructional Strategies:**

- Teach explicitly how to read and write decimals to the tenths (.1) and hundredths (.01).
- Teach explicitly the relative position of a number to the decimal point and its place value.
- Task Analysis for decimals (tenths)
  - Present a 1X10 grid and ask the student how many boxes make up the grid.
  - Shade a tenth and ask how many boxes are shaded (i.e., 1 out of 10).
  - Ask the student to write or select a written form for the decimal for 1 out of 10.
  - Ask the student to read or select a recording of the decimal.
  - Complete for multiple decimals (.1 - .9).
- Task Analysis for decimals (hundredth)
  - Present a 10X10 grid and ask the student how many boxes make up the grid.
  - Shade one hundredth and ask how many boxes are shaded (i.e., 1 out of 100).
  - Ask the student to write or select a written form for the decimal for 1 out of 100.
  - Ask the student read or select a recording of the decimal.
  - Complete for multiple decimals (.01 - .99).
- Use Model-Lead-Test\*
- Match, write or say decimals that correspond to combinations of dollars and cents.

- Student will read a money amount card. (The amount could be written as a decimal, with words, or using a cent sign.)
- Ask student to describe the money amount in another form (e.g., as a decimal).
- Use number cards to organize a stated number with a decimal.

**Suggested Supports and Scaffolds:**

- 10X10 grid paper
- Assistive technology
- Visual representations through pictures, cards, etc.
- Number line
- Place value charts
- Word cards, number cards, and grid cards for the same decimals (e.g., one tenth, .1, and a model)

**CCSS:** 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*

**CCC:** 5.NO.1c1 Rewrite a fraction as a decimal

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *M.NO.1c Using a variety of fractional and decimal representations and locating them on a number line*

**Concrete Understandings:**

- Identify some subset of item divided into ten (show me 3 of these 10ths; have 3/10 and .3 present but student is not required to read).
- Distinguish between a fraction and a decimal.

**Representation:**

- Understand place value (tenths and hundredths).
- Understand how to use a decimal point.
- Understand the following concepts, symbols, and vocabulary: equal, decimal point, tenths, and hundredths.

**Suggested Instructional Strategies:**

- Teach explicitly that tenths have one digit after the decimal point (one digit to the right of the decimal point) and hundreds have two digits after the decimal point (two digits to the right of the decimal point).
- Self-checking strategies using a calculator
  - Divide the top of the fraction by the bottom and read off/record the answer.
- Connect fractions to decimals as it relates to a dollar bill (e.g.,  $\frac{1}{2}$  of a dollar is .50 and model using a 10X10 grid).
  - Reinforce how a decimal amount and the equivalent fraction can be described in different ways of dividing a dollar. Use plastic coins to demonstrate the decimal amounts each person would receive in different situations and tell the equivalent fractional part of the dollar each person could receive.
  - Begin with most common fractions (e.g.,  $\frac{1}{2} = .5$ ,  $\frac{1}{4} = .25$ ,  $\frac{3}{4} = .75$ ).

**Suggested Supports and Scaffolds:**

- Place value chart
- Grids (with or without raised lines)
- Grids with corresponding decimal number lines
- Calculator
- Manipulatives such as base ten blocks to provide a visual representation (e.g. 8/10 is the same as .8 when represented with base ten blocks)
- Assistive technology

- Interactive whiteboard

**CCSS:** 6.RP.3c Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

**CCC:** 6.NO.1f1 Find a percent of a quantity as rate per 100

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Representing

**Progress Indicator:** *M.NO.1f Recognizing equivalence of representations using fractions, decimals, and percentages and using them solve ratio problems*

**Concrete Understandings:**

- State a relationship to a quantity out of 100 (may need to be very small numbers and be concrete, e.g., Selects 3 from an object bundle of 100).

**Representation:**

- Understand that decimals to the hundredths can be converted to percentages.
- Understand the following concepts, symbols, and vocabulary: ratio, equivalent, percent, percentage.

**Suggested Instructional Strategies:**

- Teach a problem-solving strategy to first find the whole, part (unknown), and then percent (e.g., Percentages can be thought of as rates per 100. We want to purchase something that originally cost \$12.00 but has been reduced by 25%. We know the whole is \$12.00. We want to know how much we can take off the \$12.00—this is the part or the unknown. We know that it is equal to 25%. 25% of \$12.00 can be solved by writing the percent in hundredths and then multiplying by the quantity.  $25/100 \times 12 = 4$ . 25% of \$12.00 is \$4.00).
- Teach explicitly three ways of expressing percent (e.g., 10 percent, 10%, 10/100).
- Connect fractions to decimals to percentages, as it relates to a dollar bill.
- Begin with most common fractions/percentages.
- Teach percent as “per hundred.”

**Suggested Instructional Strategies:**

- Teach a problem-solving strategy to first find the whole, part (unknown), and then percent (e.g., Percentages can be thought of as rates per 100. We want to purchase something that originally cost \$12.00 but has been reduced by 25%. We know the whole is \$12.00. We want to know how much we can take off the \$12.00—this is the part or the unknown. We know that it is equal to 25%. 25% of \$12.00 can be solved by writing the percent in hundredths and then multiplying by the quantity.  $25/100 \times 12 = 4$ . 25% of \$12.00 is \$4.00).
- Teach explicitly three ways of expressing percent (e.g., 10 percent, 10%, 10/100).
- Connect fractions to decimals to percentages, as it relates to a dollar bill.

- Begin with most common fractions/percentages.
- Teach percent as “per hundred.’
- Dual number line: For percent problems with one part missing, one side of the line is marked with the quantities and the other with the percentages; student organizes the given information and shows which information is missing.
- Bundles of 10s and 100s
- Number line
- Hundreds chart
- Calculator
- Student may use some form of graphics to represent manipulative objects, such as tally marks, asterisks, etc. Students are to place a tally mark in the appropriate location for every object that is to be recorded. Then, they represent the corresponding numerical sentence to the representation.
- Assistive Technology
- Blocks representing 10s and 100s
- Magnetic, paper, and virtual money representing 10s and 100s
- Teach using a real-world application.

**CCSS:** 3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**CCC:** 3.SE.1g1 Use  $=$ ,  $<$ , or  $>$  to compare 2 fractions with the same numerator or denominator.

**Strand:** Symbolic Expression

**Family:** Determining Equivalency

**Progress Indicator:** *E.SE.1g Using symbols ( $=$ ,  $>$ ,  $<$ ) to compare whole numbers, fractions, or decimals; write equations; and express inverse or related operations*

**Concrete Understandings:**

- Understand the concept of a fraction (a fraction is less than a whole).
- Understand the concept of comparison (greater than, less than, equal).
- Use concrete representation to determine if a fraction is equal, greater than, less than, another fraction with either the same numerator or denominator (e.g., divide a rectangle into fourths and compare  $\frac{1}{4}$  to  $\frac{3}{4}$ , break into 4 equal parts...which is more the 3 parts or the 1 part?).

**Representation:**

- Apply understanding of the symbols of  $<$ ,  $>$ , and  $=$  with whole numbers.
- Label pictorial representations of fractions, numerator, denominator.

**Suggested Instructional Strategies:**

- Multiple exemplars for equal, greater than, less than\*
- Explicit teaching of the rules of denominator and numerator
- Explicit teaching of comparisons (more of the same size parts, same number of parts but different sizes, more and less than  $\frac{1}{2}$  or 1 whole, distance from  $\frac{1}{2}$  or 1 whole)
- Choose your answer, explain why you chose the answer, test your answer

**Supports and Scaffolds:**

- Number line with fractions
- Illustrations
- Interactive whiteboard
- Computer software
- Examples of illustrations to show greater than less than, or equal
- Manipulatives



**CCSS:** 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**CCC:** 4.SE.1g2 Use  $=$ ,  $<$ , or  $>$  to compare 2 fractions (fractions with a denominator of 10 or less).

**Strand:** Symbolic Expression

**Family:** Determining Equivalency

**Progress Indicator:** *E.SE.1g Using symbols ( $=$ ,  $>$ ,  $<$ ) to compare whole numbers, fractions, or decimals; write equations; and express inverse or related operations*

**Concrete Understandings:**

- Understand the concept of a fraction (i.e., Describe that the denominator of a fraction represents the number of equal parts within a whole (length unit or region).
- Understand the concept of comparison (greater than, less than, equal).
- Identify concrete representation of a fractional part of a whole as greater than, less than, equal to another (e.g., divide a rectangle into fourths and compare  $\frac{1}{4}$  to  $\frac{3}{4}$ ; Can do by showing with parts of the whole (so relates to fractions; e.g., break into 4 equal parts...which is more the 3 parts or the 1 part?).

**Representation:**

- Apply understanding of the symbols of  $<$ ,  $>$ , and  $=$  with whole numbers.
- Label pictorial representations of fractions.

**Suggested Instructional Strategies:**

- Multiple exemplars for equal, greater than, less than\*
- Explicit teaching of the rules of denominator and numerator
- Explicit teaching of comparisons (more of the same size parts, same number of parts but different sizes, more and less than  $\frac{1}{2}$  or 1 whole, distance from  $\frac{1}{2}$  or 1 whole)
- Choose your answer, explain why you chose the answer, test your answer

**Supports and Scaffolds:**

- Number line with fractions
- Illustrations
- Interactive whiteboard
- Computer software
- Examples of illustrations to show greater than less than, or equal
- Manipulatives

**CCSS:** 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- a) Represent a fraction  $1/b$  on a number line diagram by defined the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line.
- b) Represent a fraction  $a/b$  on a number line diagram by marking off a lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number line.

**CCC:** 4.NO.116 Locate fractions on a number line

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *E.NO.11 Identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers*

**Concrete Understandings:**

- Show understanding of how parts of a whole can be expressed as fractions using numbers.
- Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts.
- Understand a fraction  $a/b$  as the quantity formed by a parts of size  $1/b$ .

**Representation:**

- Use a number line to locate whole numbers.
- Demonstrate understanding of how the numerator and denominator each influence the placement of a fraction on a number line ( $\frac{\_}{\_}$ ).

**Suggested Instructional Strategies:**

- Use sentence strips or string to fold to create their own number line.
- Explicitly teach that the denominator is the number of equal sections between 0 and 1.
- Explicitly teach that the numerator is the number of equal sections from 0, e.g. -  $3/5$  means the space between 0 and 1 has 5 equal sections and  $3/5$  is at the end of the 3rd section from zero.
- Time delay\*

**Supports and Scaffolds:**

- Interactive whiteboard
- Computer software
- Assistive Technology
- Number line with raised markers

**CCSS:** 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- a) Represent a fraction  $1/b$  on a number line diagram by defined the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line.
- b) Represent a fraction  $a/b$  on a number line diagram by marking off a lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number line.

**CCC:** 4.NO.117 Order fractions on a number line

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *E.NO.11 Identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers*

**Concrete Understandings:**

- Show understanding of how parts of a whole can be expressed as fractions using numbers.
- Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts.
- Understand a fraction  $a/b$  as the quantity formed by a parts of size  $1/b$ .

**Representation:**

- Use a number line.
- Demonstrate understanding of how the numerator and denominator each influence the placement of a fraction on a number line ( $\frac{\_}{\_}$ )
- ( $\frac{\_}{\_}$ )
- Understand the following concepts, symbols, and vocabulary: “greatest to least” and “least to greatest.”

**Suggested Instructional Strategies:**

- Folding sentence strip or paper to have students generate a number line
- Use fraction cards to place and order on a number line.
- Explicitly teach that the denominator is the number of equal sections between 0 and 1.
- Explicitly teach that the numerator is the number of equal sections from 0, e.g. -  $3/5$  means the space between 0 and 1 has 5 equal sections and  $3/5$  is at the end of the 3rd section from zero.

**Supports and Scaffolds:**

- Start with 3<sup>rd</sup> grade concept of only ordering fractions with same numerator and same denominator Interactive whiteboard
- Computer software
- Assistive Technology
- Number line with raised markers

**CCSS:** 3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

- a) Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- b) Recognize and generate simple equivalent fractions, e.g.,  $1/2 = 2/4$ ,  $4/6 = 2/3$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- c) Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form  $3 = 3/1$ ; recognize that  $6/1 = 6$ ; locate  $4/4$  and 1 at the same point of a number line diagram.*
- d) Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

4.NF.1 Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**CCC:** 4.NO.1m1 Determine equivalent fractions.

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *E.NO.1m Composing and representing equivalent fractions in the form  $a/b$*

**Concrete Understandings:**

- Describe equivalency with whole numbers (what is and what is not equivalent; this may begin with numbers/sets of objects: e.g.,  $3=3$ ).
- Describe a model of part of a whole as a fraction.
- Identify two equivalent fractions, both represented either symbolically (i.e., in numbers) (e.g.,  $1/2 = 2/4$ ) or as a model (i.e., picture) (e.g., pizza cut in halves and in fourths with  $1/2$  and  $2/4$  shaded in). If using symbolic representations, limit denominators to 2, 4, and 8 and numerators to 1-7. Use a real-world context (e.g., pizza).

**Representation:**

- Represent fractions using shaded grids by generating pictorial representations (shading circles, or rectangles, drawing on graph paper, etc.).
- Understand the following concepts, symbols and vocabulary: fraction, equivalent fractions, numerator, denominator.

**Suggested Instructional Strategies:**

- Teach equivalency explicitly using bars of equal length with the same shaded amount (e.g., show that for bars of the same length, 1 part out of 2, two parts out of 4, and 3

parts out of six, are equal (the same amount of the bar is shaded broken into 1, 2 or 3 parts)).

- Teach equivalency explicitly by using bars (visual) to show that when both the numerator and the denominator are multiplied by the same “non-zero” number, the fractions remain equivalent (e.g., to remain equal, you will always multiply or divide by 1 represented in the form of a fraction ( $\frac{2}{2}$ )).
- Teach equivalency by folding paper to create number lines- fold 2 pieces of paper the same length, fold one in half, one in fourths. Examine that  $\frac{1}{2}$  and  $\frac{2}{4}$  are the same distance from 0.
- Task analysis: Comparing fractions equal to  $\frac{1}{2}$ 
  - Present fraction bars of equal lengths that are divided into different numbers of parts with half of the parts shaded.
  - Write a fraction for each fraction bar.
  - Write a chain of equivalent fractions:  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ .
  - Then, complete a similar activity using two bars with same amount shaded (more or less than **half** of the parts).
  - Write a fraction for each fraction bar ( $\frac{2}{3}$ ,  $\frac{4}{6}$ ,  $\frac{8}{12}$ ).
  - Write a chain of equivalent fractions ( $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$ ).
- Task Analysis: Making equivalent fractions
  - Provide students with a candy bar or some representation divided into 12 parts.
  - Have them compare the division of the whole candy to a fraction the teacher provides.
  - Discuss the number of pieces of the candy that make up its given fraction.
  - Have students explain or demonstrate the meaning of equivalent fractions.
  - Use the representation selected to demonstrate equivalent fractions.
  - Students can make up new scenarios using other “wholes” that can be divided (a set of cards, a package of crackers, etc.).
- Task Analysis: Splitting bars to create equivalent fractions
  - Present a shaded fraction bar (e.g., 4 parts with 2 parts shaded ( $\frac{2}{4}$ )).
  - Write/build the numeric fraction (e.g.,  $\frac{2}{4}$ ).
  - Split each part in half, doubling the 4 parts to 8 parts doubles the shaded parts from 2 to 4).
  - Write/build the numeric new fraction (e.g.,  $\frac{4}{8}$ ).
  - Write/build a numeric chain of equivalent fractions ( $\frac{2}{4} = \frac{4}{8}$ ).
  - Explicitly state that when the numerator is doubled, by doubling the denominator, the fractions are equal.
  - Provide additional examples to show that by splitting the bar, increasing all parts of the bars increases the number of shaded parts.
- Use Model-Lead-Test\*
- Multiple exemplars (e.g., “These fractions are equivalent. These fractions are equivalent. These fractions are not equivalent.”) \*

#### **Suggested Supports and Scaffolds:**

- Assistive Technology

- Virtual bars or tiles
- Pictures that have been divided
- Geoboards
- Dot-paper
- Cuisenaire rods
- Color tiles
- Pattern blocks or sets of objects
- Pie diagrams
- Fraction bars that are ruled into certain fixed partitions and lined up for comparisons
- Multiplication tables (e.g., 1 to 4 has the same relationship as 2 to 8)

**CCSS:** 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**CCC:** 4.NO.1n2 Compare 2 given fractions that have different denominators.

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *E.NO.1n Comparing and modeling fractions, including with different denominators*

**Concrete Understandings:**

- Understand the concept of equivalency (what is and what is not equivalent; this may begin with numbers/sets of objects: e.g.,  $3 > 2$ ,  $1 < 2$ ,  $10 + 5 = 15$ ).
- Understand the concept of fraction (part of a whole) (i.e., Describe that the denominator of a fraction represents the number of equal parts within a whole (length unit or region).

**Representation:**

- Given a visual fraction model (region or number line), write the fraction with the correct numerator and denominator.
- Determine equivalent fractions (e.g.,  $\frac{1}{2} = \frac{2}{4}$ ).

**Suggested Instructional Strategies:**

- Compare fractions represented with models (e.g., circle divided in halves and in fourths with  $\frac{1}{2}$  and  $\frac{3}{4}$  shaded in).
- Use rectangles that are the same size for students to partition and represent fractions.
- Use sentence strips/paper to generate number lines.

**Supports and Scaffolds:**

- Assistive Technology
- Virtual bars or tiles
- Pictures that have been divided
- Geoboards
- Dot-paper
- Cuisenaire rods
- Color tiles
- Pattern blocks or sets of objects
- Pie diagrams
- Fraction bars that are ruled into certain fixed partitions and lined up for comparisons



**CCSS:** 3.NF.1 Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by a parts of size  $1/b$

4.NF.3 Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .

- a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- b) Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ .

**CCC:** 4.NO.2g1 Using a representation, decompose a fraction into multiple copies of a unit fraction (e.g.,  $3/4 = 1/4 + 1/4 + 1/4$ ).

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *E.NO.2g Recognizing fractions as one number/one quantity, rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions*

**Concrete Understandings:**

- Select, from given models, a model showing halves, thirds, or fourths.
- Using models demonstrate an understanding that the same denominator means equally sized portions.

**Representation:**

Given a visual fraction model (region or number line), write the fraction with the correct numerator and denominator.

**Suggested Instructional Strategies:**

- Teach explicitly using manipulatives that can be partitioned into equal sections.
- Use a number line to model decomposing fractions.
- Use graph paper or rectangles/circles for pictorial representations.
- Task Analysis: Splitting bars to create equivalent fractions
  - Present a shaded fraction bar (e.g., 4 parts with 2 parts shaded ( $2/4$ )).
  - Write/build the numeric fraction (e.g.,  $2/4$ ).
  - Split each part in half, doubling the 4 parts to 8 parts doubles the shaded parts from 2 to 4
  - Write/build the numeric new fraction (e.g.,  $4/8$ ).
  - Write/build a numeric chain of equivalent fractions ( $2/4 = 6/8$ ).
  - Explicitly state that when the numerator is doubled, by doubling the denominator, the fractions are equal.

- Provide additional examples to show that by splitting the bar, increasing all parts of the bars increases the number of shaded parts.
- Use Model-Lead-Test\*
- Multiple exemplars (e.g., “These fractions are equivalent. These fractions are equivalent. These fractions are not equivalent.”) \*

**Supports and Scaffolds:**

- Assistive technology
- Manipulatives
- Interactive whiteboard
- Computer software
- Color tiles
- Materials with raised edges

**CCSS:** 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

**CCC:** 5.NO.1c1 Rewrite a fraction as a decimal.

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *M.NO.1c Using a variety of fractional and decimal representations and locating them on a number line*

**Concrete Understandings:**

- Identify some subset of item divided into ten (show me 3 of these 10ths; have 3/10 and .3 present but not required to read).
- Distinguish between a fraction and a decimal.

**Representation:**

- Understand place value (tenths and hundreds).
- Understand decimal point.
- Understand the following concepts, symbols, and vocabulary: equal, decimal point, tenths, and hundredths.

**Suggested Instructional Strategies:**

- Teach explicitly that the tenths place is the first digit after the decimal point and hundredths place is the second digit after the decimal point.
- Time delay\*
- Self-checking strategies using a calculator
  - Divide the top of the fraction by the bottom, and read off/record the answer
- Connect fractions to decimals as it relates to a dollar bill (e.g.,  $\frac{1}{2}$  of a dollar is .50 and model using a 10X10 grid).
  - Reinforce how a decimal amount and the equivalent fraction can be described in different ways of dividing a dollar. Use plastic coins to demonstrate the decimal amounts each person would receive in different situations and tell the equivalent fractional part of the dollar each person could receive.
  - Begin with most common fractions (e.g.,  $\frac{1}{2} = .5$ ,  $\frac{1}{4} = .25$ ,  $\frac{3}{4} = .75$ ).

**Suggested Supports and Scaffolds:**

- Place value chart
- Grids (with or without raised lines)
- Grids with corresponding decimal number lines
- Calculator
- Manipulatives such as base ten blocks to provide a visual representation (e.g. 8/10 is the same as .8 when represented with base ten blocks)
- Assistive Technology

- Interactive whiteboard

**CCSS:** 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as  $\frac{62}{100}$ ; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

**CCC:** 5.NO.1c2 Rewrite a decimal as a fraction.

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *M.NO.1c Using a variety of fractional and decimal representations and locating them on a number line*

**Concrete Understandings:**

- Demonstrate an understanding of place value by identifying tenths and hundredths.
- Understand the concept of equivalency (what is and what is not equivalent; this may begin with numbers/sets of objects: e.g.,  $3 > 2$ ,  $1 < 2$ ;  $5 + 10 = 15$ ).

**Representation:**

- Know the following vocabulary: fraction ( $\frac{a}{b}$ ), decimal (.a), tenths place (.a), hundredths place (.aa)

**Suggested Instructional Strategies:**

- Explicitly teach using a task analysis.
  - Write down the decimal.
  - Divide the decimal by 1 (e.g.,  $\frac{.50}{1}$ ).
  - Multiply top and bottom by 100 (e.g.,  $\frac{50}{1} \times \frac{100}{100} = \frac{50}{100}$ ).
  - Simplify the fraction (e.g.,  $\frac{50}{100}$ , divided by the greatest common factor which is 50.  $\frac{50}{100} \div \frac{50}{50} = \frac{1}{2}$ ).
  - Teach using Least-to-most prompting. \*

**Supports and Scaffolds:**

- 10x10 decimal grid
- Assistive Technology
- Interactive whiteboard
- Computer software
- Calculator
- Multiplication and division tables

**CCSS:** 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ .*

**CCC:** 5.NO.2c2 Solve word problems involving the addition, subtraction, multiplication, or division of fractions.

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Problem Solving

**Progress Indicator:** *NO.1c Using a variety of fractional and decimal representations and locating them on a number line*

**Concrete Understandings:**

- Understand that the numerator tells the number of parts and the denominator tells the type of parts (e.g., fourths, halves).
- Identify what actions to take given the context and language used in the problem (e.g., “in all” means we add, “left” means we subtract).
- Build models to match fractions in each equation (e.g.,  $1/3 + 2/3 =$ , student will build model of each fraction).

**Representation:**

- Identify key information in a word problem to represent the total and fraction.
- Solve fraction problems using:
  - Picture
  - Models
  - Representation cards
  - Number sentences
  - Mathematical word problems
  - Graphic representation
  - Understand the following concepts, symbols, and vocabulary: +, -, X, ÷.

**Suggested Instructional Strategies:**

- Teach explicitly how to express a verbal description of a fraction (“one-fourth” as  $1/4$ ).
- Task analysis:
  - Highlight/circle important words.
  - Choose the correct operation (+, -, x, ÷).
  - Compute the answer.
  - State the answer.
- Teach explicitly how to represent the total number of objects in a word problem as an array by creating sets based on the denominator of the provided fraction in a word problem (e.g.,  $1/2$  of the 20 students would be a group of 20 objects shown as two arrays of 10 each).
- Teach explicitly how to use a number line/conversion tables to solve a word problem.
- Use Model-Lead-Test.\*

- Give students problems to model such as these: Charlene ate  $\frac{1}{4}$  of the sandwich at breakfast and  $\frac{2}{4}$  of the sandwich at lunch. How much of the sandwich did she eat?

**Suggested Supports and Scaffolds:**

- Use arrays to represent the denominator as sets.
- Number line
- Objects to represent arrays and perform operation
- Rectangular blocks engraved with dots (can be used to teach students who have visual impairment)
- Fraction strips
- Assistive Technology
- Use adapted text for word problems

**CCSS:** 6.RP.3a Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

**CCC:** 6.NO.1f4 Find a missing value (representations, whole numbers, common fractions, decimals to hundredths place, percent) for a given ratio.

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *M.NO.1f Recognizing equivalence of representations using fractions, decimals, and percentages and using them solve ratio problems*

**Concrete Understandings:**

- Create a concrete representation of a ratio (3:6 or 3 red cubes for every 6 white cubes).
- Recognize the meaning of the placement of numbers in a proportion for a given situation (e.g., If there are 11 girls in a group and 6 boys and the question is what is the ratio of boys to girls then the ratio is 6:11 because the words had boys first).

**Representation:**

- Create a table showing equivalent ratios based on a given ratio.

Inches of snow	hours
2	1
4	2
6	?
?	4

- Vocabulary
  - ratio (e.g., 2:1, 1:1)
  - percent

**Suggested Instructional Strategies:**

- When looking at a table, identify the pattern in each column.
- Teach explicitly how-to problem solve for proportional relationships:
  - Use real-life contexts such as recipes, piano keys (black to white); provide real life objects.
  - Draw pictures and use tables to determine ratios.
  - Scale up the ratio (e.g., 2:3) in fraction form ( $\frac{2}{3}$ ) to another denominator ( $\frac{x}{6}$ ).
  - Using proportional reasoning (e.g., 6 cups of flour is  $2 \times 3$  (flour), so I can multiply  $2 \times 1$  (sugar) to find the number of cups of sugar given that the original proportion is 3 cups flour to one cup sugar).



**Supports and Scaffolds:**

- Calculator
- Table of values
- Counters or cubes
- Graphic organizers
- Use real-life contexts such as recipes, piano keys (black to white) that are relevant to students.
- Draw pictures and use tables to determine proportions.

**CCSS:** 7.RP.2 Recognize and represent proportional relationships between quantities.

- a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c) Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the as  $t = pn$ .*
- d) Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

**CCC:** 7.NO.2f1 Identify the proportional relationship between two quantities.

**Strand:** Number Operations (Fractions/ratios/Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *M.NO.2f Describing proportional relationships and solving related problems*

**Concrete Understandings:**

- Recognize the constancy of one object to its parts (i.e., one face: two eyes).
- Recognize the meaning of the placement of numbers in a proportion for a given situation.
- Represent the proportion of objects (e.g., red hats) to the total number of objects (red and green hats).
- Identify the proportional relationship using visuals.

**Representation:**

- Recognize the meaning of the placement of numbers in a proportion for a given situation.
- Represent the proportion of objects (e.g., female students) to the total number of objects (students in class) (part-to-whole).
- Represent the proportion of the number of one object (female students) to the number of other objects (male students) from a set of objects (male and female students) (part-to-part).
- Find a percent of a quantity as a rate per 100 (e.g., 20% of a quantity means 20/100 or .20 times the quantity).
- Understand the following concept, symbols, and vocabulary: proportion, ratio, rate, prices, portions per person.

**Suggested Instructional Strategies:**

- Multiple Exemplar Training\*
  - Example: There are three chairs for one/each table. The ratio is **3 to 1**. The ratio is **3:1**. The ratio is **3/1**. The ratio is not **1 to 3**. Show me the proportion/ratio for three chairs for one table.
- Teach explicitly three ways to represent a proportion.

- Teach explicitly how-to problem solve for proportional relationships:
  - Use real life contexts such as recipes, piano keys (black to white); provide real-life objects.
  - Draw pictures and use tables to determine ratios.
  - Scale up the ratio (e.g., 2:3) in fraction form ( $\frac{2}{3}$ ) to another denominator ( $\frac{x}{6}$ ).
  - Using proportional reasoning (e.g., 6 cups of flour is  $2 \times 3$  (flour), so I can multiply  $2 \times 1$  (sugar) to find the number of cups of sugar given that the original proportion is 3 cups flour to one cup sugar).
- Provide a ratio and ask the student to use unit blocks to show the ratio (e.g., the ratio of girls to boys in our class is 3:2. Use the unit blocks to show the ratio of girls to boys.).
- Teach explicitly how to recognize what a proportional relationship looks like and a representation that describes the situation.

**Suggested Supports and Scaffolds:**

- Use real-life contexts such as recipes and piano keys (black to white) that are relevant to students.
- Draw pictures and use tables to determine proportions.
- Unit blocks
- Graphs, tables, equations, diagrams, and tables to show ratio
- Assistive Technology

**CCSS:** 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

**CCC:** 7.NO.2f2 Determine if two quantities are in a proportional relationship using a table of equivalent ratios or points graphed on a coordinate plane.

**Strand:** Number Operations (Fractions, Ration, Proportions)

**Family:** Determining Equivalency

**Progress Indicator:** *M.NO.1f Recognizing equivalence of representations using fractions, decimals, and percentages and using them solve ratio problems*

**Concrete Understandings:**

- Recognize the meaning of the placement of numbers in a proportion for a given situation.
- Represent the proportion of a subgroup of objects (e.g., red hats) to the total number of objects (red and green hats).
- Use a table with visuals or objects to represent proportions to determine if two numbers (i.e., 10:1) are the same proportional relationship as previous numbers (2:1, and 4:2).

**Representation:**

- Know the following vocabulary: ratio (e.g., 2:1, 1:1), equivalent, coordinate plane

**Suggested Instructional Strategies:**

- Use counters or objects to demonstrate a proportion.
- Generate a graph of values that are proportional.
- Teach skill using a variety of context (e.g., measurement, prices, pizza slices per person).

**Supports and Scaffolds:**

- Calculator
- Assistive Technology
- Interactive whiteboard
- Computer software
- Real world/meaningful context

**CCSS:** 4.NF.3 Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .

- a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ .

**CCC:** 4.NO.2h1 Add and subtract fractions with like denominators of (2, 3, 4, or 8).

**Strand:** Number Operations (Fractions, Ratios, Proportions).

**Family:** Performing Operations

**Progress Indicator:** *E.NO.2h Adding, subtracting, and multiplying fractions, including mixed numbers*

**Concrete Understandings:**

- Differentiate between parts of a fraction and the whole.
- Count the parts represented by the numerator.
- Recognize like denominators (e.g., recognize that the '4' in  $2/4$  is the same as the '4' in  $1/4$ ).
- Create a pictorial or concrete representation (using fraction strips or tiles) of fractions.
- Determine whether to use addition and subtraction strategies based on the context of the problem.

**Representation:**

- Understand the components of a fraction (numerator and denominator).
- Find the sum of two numbers.
- Vocabulary: numerator, denominator

**Suggested Instructional Strategies:**

- Model-Lead-Test\*
- Teach explicit rules for adding and subtracting fractions.
- Pizza Fractions: Cut 'pizza' circles the same size then cut them into a variety of fractions and use them to add/subtract mixed numbered fractions (e.g., add one half pizza to two  $1/4$  pieces to make a whole or subtract  $1/3$  pizza from  $6/6$ ).

**Supports and Scaffolds:**

- Fraction strips
- Fraction tiles
- Pictorial representations where the wholes are the same size
- Calculator
- Assistive Technology
- Interactive whiteboard
- Computer software

- Pattern blocks or sets of objects

**CCSS:** 4.NF.3 Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .

- a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ .

**CCC:** 4.NO.2h2 Add and subtract fractions with like denominators (2, 3, 4, or 8) using representations.

**Strand:** Number Operations (Fractions, Ratios, Proportions).

**Family:** Performing Operations

**Progress Indicator:** *E.NO.2h Adding, subtracting, and multiplying fractions, including mixed numbers*

**Concrete Understandings:**

- Differentiate between parts of a fraction and the whole.
- Count the parts represented by the numerator.
- Recognize like denominators (e.g., recognize that the '4' in  $2/4$  is the same as the '4' in  $1/4$ ).
- Create a pictorial or concrete representation (using fraction strips or tiles) of fractions.
- Determine whether to use addition and subtraction strategies based on the context of the problem.

**Representation:**

- Understand the components of a fraction (numerator and denominator).
- Find the sum of two numbers.
- Know the following vocabulary: numerator, denominator.

**Suggested Instructional Strategies:**

- Model-Lead-Test using representations\*
- Teach explicit rules for adding and subtracting fractions.
- Pizza Fractions: Cut 'pizza' circles the same size then cut them into a variety of fractions and use them to add/subtract mixed numbered fractions (e.g., add one half pizza to two  $1/4$  pieces to make a whole or subtract  $1/3$  pizza from  $6/6$ ).

**Supports and Scaffolds:**

- Fraction strips
- Fraction tiles
- Pictorial representations where the wholes are the same size
- Calculator
- Assistive Technology
- Interactive whiteboard

- Computer software
- Pattern blocks or sets of objects
- Representations of fractions with raised sections



**CCSS:** 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**CCC:** 5.NO.2c1 Solve 1 step problems using decimals.

**Strand:** Number Operations (Fractions, Ratios, Proportions).

**Family:** Performing Operations

**Progress Indicator:** *M.NO.2c Using operations and standard algorithms with whole numbers, fractions (unlike denominators), and decimals (to hundredths)*

**Concrete Understandings:**

- Given a real-world context, determine when to add, subtract, multiply, and divide.
- Understand that numbers to the right of the decimal represent a value less than one.
- Follow rules for decimal point placement when adding, subtracting, multiplying, or dividing.

**Representation:**

- Understand symbols for +, -, ×, ÷
- Know the following vocabulary: decimal point, decimal.

**Suggested Instructional Strategies:**

- Teach problem solving strategies to determine operations.
- Use task analytic instruction to teach steps to solve word problems.
- Teach using Least to Most prompts\*
- Use Model-Lead-Test\*
- Have students self-check their answers. Start by modeling this process.
- To demonstrate addition, gather several representations labeled with the decimal (circles, squares, pattern blocks, Cuisenaire rods) and identify how many of the pieces make one whole (e.g.,  $.5 + .5$ ).

**Supports and Scaffolds:**

- 10x10 hundreds of grids
- Place value chart
- Calculator
- Assistive Technology
- Interactive whiteboard
- Computer software

**CCSS:** 6.NS.1 Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi? Compute fluently with multi-digit numbers and find common factors and multiples.*

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

**CCC:** 6.NO.2c3 Solve one step, addition, subtraction, multiplication, or division problems with fractions or decimals.

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Performing Operations

**Progress Indicator:** *M.NO.2c Using operations and standard algorithms with whole numbers, fractions (unlike denominators), and decimals (to hundredths)*

**Concrete Understandings:**

- Understand and apply the concept of addition, subtraction, multiplication, and division.
- Identify a fraction and decimal.
- Given a context, choose the correct operation (e.g., altogether, take away).

**Representation:**

- Relates fractions and decimals to pictorial representations.
- Understand the following symbols, concepts and vocabulary: +, -,  $\times$ ,  $\div$ , fraction and decimal ( $a/b$ ,  $a$ ).

**Suggested Instructional Strategies:**

- Use multiple exemplar training to teach part to whole. \*
- Task analysis\*
- Teach using Least Intrusive Prompts. \*
- Use Model-Lead-Test.\*
- Have students self-check their answers. Start by modeling this process.
- To demonstrate addition, gather several representations of halves (circles, squares, pattern blocks, Cuisenaire rods) and identify how many of the pieces make one whole. Discuss that adding one more half makes the sum bigger than the whole.
- Present fraction and decimal patterns. For example, the denominator stays the same and the numerator increases by 1. Ask the student to find the next number in the sequence.

**Suggested Supports and Scaffolds:**

- Provide meaningful manipulatives, counters and/or picture representations with symbol included.
- Templates with formulas
- Graphic organizers (e.g., place value table) for visual structure
- Number lines
- Conversion tables
- Smart board technology
- Fraction strips
- Number lines with tactile representations
- Assistive Technology
- Large print or Braille representations of symbols and vocabulary
- Teach using a real-world application or task

**CCSS:** 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

**CCC:** 8.NO.2i4 Solve two step addition, subtraction, multiplication, and division problems with fractions, decimals, or positive/negative numbers.

**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Performing Operations

**Progress Indicator:** *M.NO.2i Using operations with rational numbers; representing rational numbers and approximations of irrational numbers on a number line*

**Concrete Understandings:**

- Understand and apply the concept of addition, subtraction, multiplication, and division.
- Identify positive and negative numbers on a number line.
- Identify a fraction and decimal.
- Given a real-world context, select the correct operations (e.g., altogether, take away).

**Representation:**

- Locate relevant information within a word problem.
- Understand the following concepts, symbols, and vocabulary: addition, subtraction, multiplication, division, positive/negative numbers, number line.

**Suggested Instructional Strategies:**

- Prime background knowledge and connections by using real-world context.
- Task analysis of steps to solve two-step word problems (this could include using Least Intrusive Prompts)
  - When solving word problems, teach how to determine which part of the problem is given and which part needs to be determined/solved
- Use Model-Lead-Test (This involves model solving a word problem while thinking out loud. For example, pointing out the key words and the operations they call for. For example, “To find the whole, we add. Then, to find a part, we subtract.” Next follow the steps as a group with the teacher leading as needed. Last, give students an opportunity to complete the steps without help.) \*
- Teach explicitly the rules for solving problems involving computation providing templates/formulas
- Model addition/subtraction equations by placing the appropriate numbers of chips on a graphic organizer. Using the notion of opposites, demonstrate how to simplify by removing pairs of opposite colored chips.

**Suggested Supports and Scaffolds:**

- Highlight text using tape, pen, computer highlighting that provide important information

- Provide visual representations (e.g., grids) of problem with symbols
- Number line
- Labeled problem
- Use graphic organizers
- Conversion tables
- Calculator
- Use number lines to present a visual image for students to visualize addition and subtraction results.
- Provide two-colors of counters or colored chips as a physical and kinesthetic model for adding and subtracting integers. Let one color represent positives and the second color negatives.
- Assistive Technology

### Equation prompt

First fact	Sign	Second fact	Sign	Last fact
	+ -		=	

A number line from 1 to 10. Above each number is a dot. Below the line, an arrow points from left to right with the word "Add" above it. Another arrow points from right to left with the word "Subtract" above it.

X = \_\_\_\_\_

**CCSS:** 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

**CCC:** 7.NO.2f6 Solve word problems involving ratios.







**Strand:** Number Operations (Fractions/Ratios/Proportions)

**Family:** Problem Solving

**Progress Indicator:** *M.NO.2f Describing proportional relationships and solving related problems*

**Concrete Understandings:**

- Given a scenario, find the 2 quantities in a ratio (e.g., “Bill has traveled 460 miles on 10 gallons of gas. What is the ratio?” What is the unit of measure- gallons of gas, miles)?
- Relate the placement of numbers in a ratio to the given context (the meaning of 46:1, 46 equals miles, 1 equals a gallon of gas).
- Use a table with visuals or objects to represent proportions to solve ratio problem.

Inches of snow	Hours
2 	1 
4 	2 
10 	1 

**Representation:**

- Locate relevant information within a word problem.
- Make sense of a word problem to create a proportion.
- Understand the following concepts and vocabulary: ratio, proportion, rate.

**Suggested Instructional Strategies:**

- Help students access background knowledge and connections by using real world context.
- Task analysis of steps to solve word problems (this could include using Least Intrusive Prompts)

- When solving word problems, teach how to determine which part of the problem is given and which part needs to be determined/solved.
- Use Model-Lead-Test\*
- Teach explicitly the rules for solving problems involving ratios providing templates/formulas.
- Use multiple exemplar training. \*
- Task Analysis example:
  - Read the story problem/situation: “In one (1) day, Jack eats three meals. How many meals will Jack eat in 5 days?”
  - Use the information to label two rows in a table. Find the word/picture that follows the number. Write the first word/picture in the first row/ column (point to the row/column).
  - Write the second word/picture in the second row/first column (point to the row/column).
  - Use the information in the problem/situation to fill in the number of days (a).
  - Use the information in the problem/situation to fill in the number of meals (b).
  - Determine the relationship between “a” and “b” ( $a \times \_ = b$ )

Day (a)	1	2	3	4	5
Total Meals (b)	3	6	9	—	—

- Here is a way to show the ratio / compare the two numbers. The first row is a and the second row are b ( $\frac{a}{b}$ ,  $5/b$ ,  $5/15$ ).
- You showed the ratio of days to meals. Show/tell me the ratio.

**Suggested Supports and Scaffolds:**

- Highlight text that provide important information.
- Provide visual representations (e.g., grids) of problem with symbols.
- Tables (vertical or horizontal) with two labeled columns/rows to illustrate the ratio (e.g., Maria stamps three letters every minute which we write as 3:1. Show me the letters she stamps in a minute.).

Stamp	Minutes
3	1
—	2
—	3

- Voice output devices or talking software
- Calculator
- Labeled problem
- Conversion table
- Use graphic organizers
- Assistive Technology
- Highlight text using tape, pen, computer highlighting