Elements of the Instructional Families:
Number and Operations – Real Numbers

Teaching Number Operations

All of the CCCs in this document relate to teaching Number Operations. Below are some additional resources that may be helpful:

Multiplication Curriculum

Math Courses

Math Games

Virtual Math Lessons

Teaching Children Math

CCC Mathematics | Number Operations (Real Numbers)

CCSS: K.CC.1 Count to 100 by ones and by tens.

CCC: K.NO.1a2 Rote count to 31.

Strand: Number Operations (Real Numbers)

Family: Counting and Representing Numbers

Progress Indicator: *E.NO.1a Showing mastery of the prerequisite core skills of cardinality, constancy, and 1:1 correspondence*

Concrete Understandings:
- Repeat a number after a teacher orally says the number.

Representation:
- Know numbers in order from 1-10.

Suggested Instructional Strategies:
- Model counting forward, have student mimic or repeat.
- Teach in small increments, adding 2-3 numbers at a time (no more than 5), starting with the number 1.
- If student misses a number, teacher stops student and models by saying four numbers beginning with the two numbers before the missed number (e.g., student misses 8 so teacher prompts with 6, 7, 8, 9), have student try to say this section of numbers (Stein, Kinder, Silbert, & Carnine, 2006).
- Response shaping (e.g., Teach "1,2,3,go!"; then "1,2,3,4 out the door!"; then "1,2,3,4,5, let's slide!"; Sadler, 2009)
- Give students a number line or a hundreds chart and have them orally count numbers as they move their finger across the number line or hundreds chart.
- Give students a container of objects and have them orally count as they place objects.
- Teach using a song, chant, or tune.

Supports and Scaffolds:
- Assistive Technology (voice output)
- Auditory cues (snap, clap…)
- Response cards with numerals

CCSS: K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

CCC: 1.NO.1a7 Count forward beginning from any given number below 10.

Strand: Number Operations (Real Numbers)

Family: Counting and Representing Numbers

Progress Indicator: *E.NO.1a Showing mastery of the prerequisite core skills of cardinality, constancy, and 1:1 correspondence*

Concrete Understandings:
- Continues rote counting after teacher starts (teacher says "2, 3…").
- Can rote count.

Representation:
Count from _ to _, with a number line as a model .

Suggested Instructional Strategies:
- Model counting forward and have student mimic or repeat.
- Teach using a song, chant, or tune.
- Teach in frequent and short drills.
- If student misses a number, teacher stops student, and models by saying four numbers beginning with the two numbers before the missed number (e.g., student misses 8 so teacher prompts with 6,7,8,9), have student try to say this section of numbers (Stein, Kinder, Silbert, & Carnine, 2006).
- Use counters. Give student 10 counters, line up counters. Have students count 3 counters and cover them with their left hand. Ask how many are under the hand, if needed teacher can say "three." Teacher says, "Count like this: 3 (pointing to the student's hand), 4, 5… Repeat with other numbers under the hand."
- Teach nonverbal students to "count in their head" and make a movement with each number or to use Assistive Technology for counting.

Supports and Scaffolds:
- When initially teaching, begin with numbers in the middle of the 'decade' of numbers, so the student doesn't have to start all the way at 1, and doesn't have only 1-2 numbers left in the decade (e.g., when asking a student to count to 31, start with 25, 26, or 27 rather than 21 or 28,29; Stein, Kinder, Silbert, & Carnine, 2006).
- Assistive Technology (voice output)
- Number line
- Manipulatives for counting
- iPad

CCSS: K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
   a) When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

CCC: 1.NO.1c1 Use a number line to count up to 31 objects by matching 1 object per number.
Strand: Number Operations (Real Numbers)

Family: Counting and Representing Numbers

Progress Indicator: *E.NO.1c Developing number line skills (linear representations) using 0 to 20, and later 0 to 100*

Concrete Understandings:
   • Correctly matches one object to one number as they count.
   • Can correctly rote count to 31.

Representation:
Recognize the numbers 1-31 on a number line.

Suggested Instructional Strategies:
   • Teach using Model-Lead-Test if student will be touching numbers and counting.
   • Teach using a system of least prompts if student will count objects.

Supports and Scaffolds:
   • Interactive whiteboards
   • A number line with a picture representation for each object to be counted, will only require the student to match
   • A number line with a raised dot paired with each number, and/or Braille
   • Provide a number line composed of raised cells

1	2	3	4	5	6	7	8	9	10	11	12

CCSS: 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a) 10 can be thought of as a bundle of ten ones – called a "ten".
   b) The numbers from 11 to 19 are composed of a ten and one, two, three four, five, six seven, eight, or nine ones.

CCC: 1.NO.1h2 Identify the value of the numbers in the tens and ones place within a given number up to 31.

Strand: Number Operations (Real Numbers)

Family: Understanding the Base Ten Number System

Progress Indicator: E.NO.1h *Applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form*

Concrete Understandings:
   • Identify a bundle of 10.
   • Group 10 ones into a bundle of 10.
   • Recognize in a 2-digit number the left most digit represents the number of groups of tens and the right most digit represents the number of ones.

Representation:
   • Recognize a set of 10 as 10 without counting.
   • Count to 3 bundles of 10 by counting orally by 10s.

Suggested Instructional Strategies:
   • Visual representation, such as a straw activity. Use a pocket chart with 10's and 1's pockets. Put a specified number of straws in the 1's pocket (e.g., 22). Have students count out 10 straws and then bundle with a rubber band and place in the 10's. For 22, students should bundle two sets of 10 and place in 10's pocket and have 2 straws left in 1's pocket.
   • Task analysis
       o Place number on graphic organizer so that the numbers are in the correct columns.
       o Match the number in the ones column to the representations.
       o Match the number in the tens column to the representation.
   • Teach with T-chart or other graphic organizer for tens and ones
   • Base ten kit
   • Ten Frame (Introducing the Ten Frame)
   • Bundle sticks (10) with rubber band

Supports and Scaffolds:
   • 100s chart
   • Visual representations
   • 10 little counters = one big counter

- Computer software
- Place value mat or pocket chart with a column for tens and a column for ones
- Provide cards with sets of tens and sets of ones that can be used to represent the values of tens and ones
- Graphic organizer or place value template

CCSS: K.NBT.1 Compose and decompose numbers from 11 to 19 into tens ones and some further ones, e.g., by using objects or drawings, and record each compositions or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: The numbers from 11 to 19 are composed of a ten and one, two, three four, five, six seven, eight, or nine ones.

CCC: 1.NO.1h1 Build representations of numbers up to 19 by creating a group of 10 and some 1s (e.g., 13 = one 10 and three 1s).

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1h Applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form*

Concrete Understandings:
- Identify a bundle of 10.
- Group 10 ones into a bundle of 10.
- Recognize in a 2-digit number the left most digit represents the number of groups of tens and the right most digit represents the number of ones.

Representation:
- Identify numbers within 19.
- Match numbers to sets of tens and ones.
- Understand the following concepts, symbols, and vocabulary: place value, tens, ones.

Suggested Instructional Strategies:
- Use a calendar with pockets to exchange (Resources for Teachers).
- Money exchange (e.g., ten pennies for a dime etc.)
- Teach using ten frame (Introducing the Ten Frame).
- Base ten kit with graphic organizer
- Task analysis (e.g., identify the number in the tens place, count out the number, identify the number in the ones place, count out the number)

Supports and Scaffolds:
- Ten frame graphic organizer with counters or unfix cubes
- Popsicle stick with 10 beans per stick for tactile representation
- Unfix® cubes
- Interactive whiteboards

CCSS: K.CC.3 Write numbers from 0 to 20. Represent several objects with a written numeral 0-20 (with 0 representing a count of no objects).

CCC: 1.NO.1i1 Recognize zero as representing none or no objects.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1i Recognizing zero as an additive identify, origin for the number line, and representing no units as a quantity or in place value*

Concrete Understandings:
- Identify a set of zero.
- Count sets that are not zero.
- Orally tell that a set of counters has more than a set with zero objects.

Representation:
Represent a set of zero with the number 0.

Suggested Instructional Strategies:
- Teach 1, 2, and 3 then go back to teaching 0 (e.g., egg carton sections).
- Movement exercises where teacher orally gives directions (e.g., jump three times, jump zero times).

Supports and Scaffolds:
- Zero in a real-world context (e.g., behavior chart 3 strikes 1, 2, 3 or zero strikes with balls).
- Students or other object (visual representation) on one side of room and zero on other side of room.

CCSS: 1.OA.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6+ 4 = 2 + 10 = 12. (Associative property of addition.)*

CCC: 1.NO.1i2 Recognize zero as an additive identity.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1i Recognizing zero as an additive identify, origin for the number line, and representing no units as a quantity or in place value*

Concrete Understandings:
- Identify a set of zero.
- Complete an addition problem using manipulatives when one value is zero.

Representation:
- Identify an equation demonstrating the additive identity.
- Apply the additive identity property to solve an equation using counters or a number line.
- Understand the following concepts, symbols, and vocabulary: zero, addition, words that mean addition (e.g., altogether, plus, total, in all), additive identity.

Suggested Instructional Strategies:
- Use memorization strategies (e.g., Always the number it says, ex. 2+0=2).
- Use number line where adding 0 means that you do not move on the number line
- Model-Lead-Test*
- "Use part-part-whole mat. For example, 3 + 0 = 3, one side would have 3 counters, one side would be blank. The student would be responsible for answering the question, "How many are there total?"
- For an example of mat visit this site

Supports and Scaffolds:
- Use student's interest for a real-world application
- Graphic organizer

CCSS: 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCC: 2.NO.1e3 Write or select the numerals 0-100.

Strand: Number Operations (Real Numbers)

Family: Counting and Representing Numbers

Progress Indicator: *E.NO.1e Reading and writing numbers; counting and estimating (e.g., how many? skip counting by 2s, 5s, 10s; even/odd)*

Concrete Understandings:
Student can write or select a given number when provided with a set of objects that matches the number.

Representation:
- Identify the numeral after a teacher model.
- Read written numbers orally in and out of sequence.

Suggested Instructional Strategies:
- Teach students to read numbers using time delay. It is recommended that you not teach numbers 1-10 in sequential order to separate similar looking (6,9 and 0,1,10) and similar-sounding (4,5) numbers. One suggested order is 4,2,6,1,7,3,0,8,5,9,10 (Stein, Kinder, Silbert, & Carnine, 2006).
- Teach students to write numerals with graduated guidance.
- Provide opportunities to become independent through the use of worksheets.

Supports and Scaffolds:
- Students should be taught to read and write numbers in increments (1-10 first, the teens second: 11-20, and then 20-99). Also, students should be able to read numbers fluently before learning to write them (Stein et al., 2006).
- Use blocks or Cuisenaire® rods and place value chart to reinforce concept of place value.
- Interactive whiteboards
- Computer software
- Strip of Velcro numbers
- Worksheet with traceable numbers
- Stencils
- Tactile numbers and/or Braille

CCSS: 2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones: e.g. 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a) 100 can be thought of as a bundle of ten tens – called a "hundred."
b) The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

CCC: 2.NO.1h5 Build representations of 3 digit numbers using hundreds, tens and ones.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1h Applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form*

Concrete Understandings:
- Identify a bundle of 10.
- Identify a bundle of 100.
- Group 10 ones into a bundle of 10.
- Group 10 tens into a bundle of 10.

Representation:
- Match numbers to bundle sets of hundreds, tens and ones.
- Understand the following concepts, symbols, and vocabulary: hundreds, tens, ones, place value.

Suggested Instructional Strategies:
- Use a base ten kit.
- Use a visual representation, such as a straw activity. Use a pocket chart with 10's and 1's pockets. Put a specified number of straws in the 1's pocket (e.g., 22). Have students count out 10 straws and then bundle with a rubber band and place in the 10's. For 22, students should bundle two sets of 10 and place in 10's pocket and have 2 straws left in 1's pocket.
- Teach using a graphic organizer (e.g., T chart hundreds, tens, and ones).
- Teach in the context of money (e.g., using dimes and pennies).
- Teach using flip cart with each section (0-9) or hundreds, tens, ones.

Supports and Scaffolds:
- Use student's interest for a real-world application.
- Visual representation using ten frames or ten set of ten counters=100.
- Graphic organizers
- Interactive whiteboard or other technology to manipulate representations

CCSS: 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCC: 2.NO.1h8 Write or select expanded form for any 2-digit number.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1h Applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form*

Concrete Understandings:
- Numbers are read from left to right.
- Use manipulatives to show the number of 10s and 1s for a given number within 99.
- Recognize in a multi-digit number the left most digit represents the number of groups of tens and the right most digit represents the number of ones.
- Recognize that a number can decomposed by place and represented as an addition equation—56 = 50 + 6.

Representation:
- Identify number in tens and ones place.
- Understand the following concepts, symbols, and vocabulary: ones, tens, place value.

Suggested Instructional Strategies:
- Place Value Mat (see example below)
- Base ten kit

Supports and Scaffolds:
- Start with color coded templates as they relate to tens and ones and remove for generalization
- Expanded form template (e.g., _____ + _____)

CCSS: 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCC: 2.NO.1h9 Write or select expanded form for any 3-digit number.

Strand: Number Operations (Real Numbers)

Family: Understanding Base Ten Number System

Progress Indicator: *E.NO.1h Applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form*

Concrete Understandings:
- Numbers are read from left to right.
- Use manipulatives to show hundreds, tens, ones for a given number within 999.
- Recognize in a multi-digit number the left most digit represents the number of groups of tens and the right most digit represents the number of ones.
- Recognize that a number can be decomposed by place and represented as an addition equation: 569 = 500 + 60 + 9.

Representation:
- Identify number in hundred's, ten's, and one's place.
- Know the value of each of the digits in a 3-digit number.
- Understand the following concepts, symbols, and vocabulary: place value, ones, tens.

Suggested Instructional Strategies:
- Place value mat (see example below)
- Base ten kit
- Model writing expanded form
- Provide worksheets for increased independence
- Provide multiple examples of 3-digit numbers paired with their expanded form

Supports and Scaffolds:
- Start with color coded templates as it relates to tens and ones and remove for generalization
- Expanded form template (e.g., _____ + _____)
- Computer software
- Place value mat with a column for hundreds, tens and ones

Hundreds	Tens	Ones

- Graphic organizer that provides the correct number of "0s"

549 = _00 +_0 + _	
• Computer software	

CCSS: 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCC: 2.NO.1i3 Explain what the zero represents in place value (hundreds, tens, ones) in a number.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1i Recognizing zero as an additive identify, origin for the number line, and representing no units as a quantity or in place value*

Concrete Understandings:
- Identify the zero in each number.
- Identify hundreds bundles, tens bundles, and ones.

Representation:
- Match appropriate bundles to given numbers.
- Explain that a zero represents none for that given place value. E.g., 304 has 3 hundreds 0 tens and 4 ones.
- Understand the following concepts, symbols, and vocabulary: place value, tens, ones, hundreds.

Suggested Instructional Strategies:
- Base ten kit
- Model-Lead-Test*
- Example, non-example (e.g., show examples of zero objects and non-examples of zero objects)

Supports and Scaffolds:
- Assistive Technology
- Place value mat or another graphic organizer

CCSS: 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCC: 3.NO.1j2 Write or select the expanded form for up to 3-digit number.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1j Applying place value concepts to read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place*

Concrete Understandings:
- Use manipulatives to show hundreds, tens, and ones, for a given number.
- Recognize that a number can decomposed by place and represented as an addition equation: 569 = 500 + 60 + 9.

Representation:
- Identify number in hundreds, tens, and ones place.
- Understand the following concepts, symbols, and vocabulary: place value, ones, tens, hundreds.

Suggested Instructional Strategies:
- Place value Mat
- Base ten kit

Supports and Scaffolds:
- Start with color coded templates as it relates to tens and ones and remove for generalization.
- Expanded form template (e.g., _____ + _____)

CCSS: 2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones: e.g. 706 equals 7 hundreds 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a) 100 can be thought of as a bundle of ten tens – called a "hundred."
b) The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

CCC: 3.NO.1j1 Build representations of numbers using hundreds, tens, and ones.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1j Applying place value concepts to read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place*

Concrete Understandings:
- Identify a bundle of ten, a bundle of one hundred, and a one.
- Count bundles of 10, count bundles of 100.
- Recognize in a multi-digit number the left most digit represents the number of groups of tens and the right most digit represents the number of ones.
- Recognize that a number can decomposed by place and represented as an addition equation: 569 = 500 + 60 + 9.

Representation:
Understand how place value works (e.g., the "3" in "34" represents 3 tens). Label a bundle of ten, a bundle of one hundred, and a one.

Suggested Instructional Strategies:
- Use a base ten kit.
- Use a visual representation, such as a straw activity. Use a pocket chart with 10's and 1's pockets. Put a specified number of straws in the 1's pocket (e.g., 22). Have students count out 10 straws and then bundle with a rubber band and place in the 10's. For 22, students should bundle two sets of 10 and place in 10's pocket and have 2 straws left in 1's pocket.
- Teach using a graphic organizer (e.g., T-chart hundreds, tens and ones).
- Teach in real-world context.
- Teach using flip cart with each section (0-9) or hundreds, tens, ones.

Supports and Scaffolds:
- Use student's interest for a real-world application.
- Visual representation using ten frames or ten set of ten counters=100
- Graphic organizers
- Interactive whiteboard or other technology to manipulate representations

CCSS: 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCC: 3.NO.1j3 Use place value to round to the nearest 10 or 100.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1j Applying place value concepts to read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place*

Concrete Understandings:
- Identify ones, tens, hundreds place.
- Use a place value chart.
- Recognize that numbers 1-4 are closer to 0 and numbers 6 through 9 are closer to 10.
- Identify 5 as a number in the middle but know that we round up.

Representation:
- Identify the nearest ten and the nearest hundred
- Understand the following concepts, symbols, and vocabulary: place value, ones, tens, hundreds

Suggested Instructional Strategies:
- Use video resources (e.g., <u>Brain Pop Jr</u>.)
- Explicit instruction of the rules
- Task analysis (e.g., if rounding to the tens place, find the ten above and below the number, use rules to determine whether to round up or down)

Supports and Scaffolds:
- Make rules available on a "cheat sheet"
- Number line
- Interactive whiteboard or other technology to manipulate representations
- Assistive Technology

CCSS: 2.NBT.4 Compare two three-digit numbers based on meanings if the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

CCC: 3.NO.1h1 Compare 3-digit numbers using representations and numbers (e.g., identify more hundreds, less hundreds, more tens, less tens, more ones, less ones, larger number, smaller number).

Strand: Number Operations (Real Numbers)

Family: Determine Relative Position of Whole Numbers

Progress Indicator: *E.NO.1h Applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form*

Concrete Understandings:
- Identify a base ten bundle (e.g., tens, hundreds).
- Using base 10s, build numbers within 999.
- Decompose into expanded form to compare.

Representation:
- Identify pictorial representation of ones, tens, hundreds.
- Understand the following concepts, symbols, and vocabulary for: <, >, =.
- Identify place or describe value of digit based on place.

Suggested Instructional Strategies:
- Task analysis (e.g., steps to place counter on number line for each number, make comparison, enter answer)
- Chunking on content (e.g., place value of hundred, tens, one; compare hundreds first (then tens, then ones); symbols for comparisons)

Supports and Scaffolds:
- Number line
- Hundreds, tens, ones blocks
- Place value template to compare each number
- Jig/mat with ten squares
- Popsicle sticks to make a bundle
- Interactive whiteboards or other technology to manipulate representations
- iPad applications
- 1000s chart

CCSS: 4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

CCC: 4.NO.1j5 Use place value to round to any place (i.e., ones, tens, hundreds, thousands).

Strand: Number Operations (Real Numbers)

Family: Understanding Base Ten Number System

Progress Indicator: *E.NO.1j Applying place value concepts to read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place*

Concrete Understandings:
- Identify ones, tens, hundreds in bundled sets
- Make comparisons between similar/different with concrete representations (i.e., is this set of manipulatives (8 ones) closer to this set (a ten) or this set (a zero)?
- Recognize that numbers 1-4 are closer to 0 and numbers 6 through 9 are closer to 10.
- Identify 5 as a number in the middle but know that we round up.

Representation:
- Identify pictorial representation of numbers in ones, tens, hundreds blocks.
- Match vocabulary of ones, tens, hundreds, thousands to digits in a number.

Suggested Instructional Strategies:
- Explicit instruction on rules for rounding using a number line
- Task analysis for rounding (e.g., circle place value, arrow next number, arrow number tells circle number what to do, make decision, enter answer)
- Model-Lead-Test*

Supports and Scaffolds:
- Number line or number chart
- Interactive whiteboards or other technology to manipulate representations
- Graphic organizer or place value template
- Apply quantities to coin values for a real-world application (e.g., 28¢ rounds up to 30¢)

CCSS: 4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

CCC: 4.NO.1j7 Write or select the expanded form for a multi-digit number.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *E.NO.1j Applying place value concepts to read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place*

Concrete Understandings:
- Identify bundles as a 1, 10, or 100.
- Identify the appropriate bundle for each digit in the multi-digit number within 999.
- Recognize in a multi-digit number the left most digit represents the number of groups of tens and the right most digit represents the number of ones.
- Recognize that a number can be decomposed by place and represented as an addition equation: 569 = 500 + 60 + 9.

Representations:
- Expanded form of number.
- Understand the following concepts, symbols, and vocabulary: ones, tens, hundreds, place value.

Suggested Instructional Strategies:
- Place value Mat
- Base ten kit

Supports and Scaffolds:
- Start with color coded templates as it relates to tens and ones and remove for generalization
- Expanded form template (e.g., _____ + _____)

CCSS: 6.NS.7c Understand ordering and absolute value of rational numbers. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars write |-30| = 30 to describe the size of the debt in dollars.*

CCC: 6.NO.1e1 Determine the meaning of absolute value.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *M.NO.1e Describing, representing, and comparing absolute value relationships*

Concrete Understandings:
- Identify value of the number and the distance of that number from zero on a number line.
- Match the positive and the negative value of the same number on the number line.

Representation:
- Identify absolute values of numbers.
- Understand the following concepts, symbols, and vocabulary: absolute value, zero

Suggested Instructional Strategies:
- Teach using a number line (distance from zero)
- Explicit instruction of the rules (e.g., always a positive number)
- Model-Lead-Test*

Supports and Scaffolds:
Real world application (e.g., if you have -30 dollars in your account you will need 30 dollars to bring your account to get you out of the negative)

CCSS: 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

CCC: 6.NO.1i1 Identify what an exponent represents (e.g., 8^3= 8X8X8).

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *M.NO.1i Using exponents and scientific notation to express very large or very small quantities*

Concrete Understandings:
- Produce the correct amount of base numbers to be multiplied given a graphic organizer or template.
- Recognize that a number with an exponent means that the base is multiplied repeatedly the number of times equal to the exponent.

Representation:
- Select the correct expanded form of what an exponent represents (e.g., 8^3=8x8x8).
- Identify the number of times the base number will be multiplied based on the exponent.
- Understand the following concepts, symbols, and vocabulary: base number, exponent

Suggested Instructional Strategies:
- Create diagram or number tree
- Task analysis
    - Identify the exponent of a number
    - Identify the number
    - Write the number, the number of times indicated by the exponent

Supports and Scaffolds:
- Circle or highlight the raised number (exponent)
- Graphic organizer

CCSS: 8.EE.3 Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*

CCC: 8.NO.1i1 Convert a number expressed in scientific notation up to 10,000.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *M.NO.1i Using exponents and scientific notation to express very large or very small quantities*

Concrete Understandings:
- Identify a base ten bundle (e.g., tens, hundreds, thousands).
- Select the appropriate base ten bundle to represent the number expressed in scientific notation.

Representation:
- Select the correct numeric representation.
- Identify the correct symbol used when converting scientific notation.
- Understand the following concepts, symbols, and vocabulary: scientific notation, base number, exponent.

Suggested Instructional Strategies:
- Task analysis
  - Identify 10 as the place value
  - Identify the exponent
  - Multiply by the coefficient
- Model-Lead-Test through the steps of the task analysis*

Supports and Scaffolds:
- Internet converters
- Graphic organizer
- Calculator
- Website support

CCSS: 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.

CCC: 8.NO.1k1 Identify π as an irrational number (e.g., not necessary for counting, computation, etc.).

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *M.NO.1k Distinguishing rational numbers (terminating and repeating) from irrational numbers (non-terminating and non-repeating), and recognizing that together they form the real number system and that both can be represented on the number line*

CCC: 8.NO.1k1 Identify π as an irrational number (e.g., not necessary for counting, computation, etc.).

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *M.NO.1k Distinguishing rational numbers (terminating and repeating) from irrational numbers (non-terminating and non-repeating), and recognizing that together they form the real number system and that both can be represented on the number line*

Concrete Understandings:
- Identify 3.14 as π.
- Understand that the use of 3.14 for π is a rounded approximated number.

Representation:
- Identify the symbol for π in writing and on a calculator.
- Understand the following concepts, symbols, and vocabulary: irrational numbers, rational numbers.

Suggested Instructional Strategies:
- Explicit instruction on rational and irrational numbers
- Multiple exemplar training of types of numbers rational and irrational*
- Video resource: Brain pop

Supports and Scaffolds:
Visual support for the symbol and vocabulary of pie

CCSS: 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.

CCC: 8.NO.1k2 Round irrational numbers to the hundredths place.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *M.NO.1k Distinguishing rational numbers (terminating and repeating) from irrational numbers (non-terminating and non-repeating), and recognizing that together they form the real number system and that both can be represented on the number line*

Concrete Understandings:
- Identify place value to the hundredths place.
- Apply the rule for rounding (e.g., find number on a number line, if 5 or greater, round up, if less than 5, round down).

Representation:
- Identify the nearest tenth and nearest hundredth.
- Understand the following concepts, symbols, and vocabulary: place value, ones, decimal, tenths, hundredths.

Suggested Instructional Strategies:
- Use video resources (e.g., Brain Pop Jr.)
- Explicit instruction of the rules
- Task analysis (e.g., if rounding to the tens place, find the ten above and below the number, use rules to determine whether to round up or down)

Supports and Scaffolds:
- Make rules available on a "cheat sheet"
- Number line
- Interactive whiteboard or other technology to manipulate representations
- Assistive technology
- Place value template

CCSS: N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

CCC: H.NO.1a3 Convert a number expressed in scientific notation.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *H.NO.1a Using exponents and scientific notation to represent quantities and expressions*

Concrete Understandings:
- Identify a base ten bundle (e.g., tens, hundreds, thousands).
- Select the appropriate base ten bundle to represent the number expressed in scientific notation.

Representation:
- Select the correct numeric representation.
- Counting out the place value to use as the exponent in writing in scientific notation.
- Understand the following concepts, symbols, and vocabulary: scientific notation, base number, exponent

Suggested Instructional Strategies:
- Task analysis
    - Identify 10 as the place value.
    - Identify the exponent.
    - Multiply by the coefficient.
    - Model-Lead-Test through the steps of the task analysis*

CCSS: N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

CCC: H.NO.1a3 Convert a number expressed in scientific notation.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *H.NO.1a Using exponents and scientific notation to represent quantities and expressions*

Concrete Understandings:
- Identify a base ten bundle (e.g., tens, hundreds, thousands).
- Select the appropriate base ten bundle to represent the number expressed in scientific notation.

Representation:
- Select the correct numeric representation.
- Counting out the place value to use as the exponent in writing in scientific notation.
- Understand the following concepts, symbols, and vocabulary: scientific notation, base number, exponent

Suggested Instructional Strategies:
- Task analysis
    - Identify 10 as the place value.
    - Identify the exponent.
    - Multiply by the coefficient.
    - Model-Lead-Test through the steps of the task analysis*

Supports and Scaffolds:
- Internet converters
- Graphic organizer
- Calculator
- Website support

CCSS: N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

CCC: H.NO.1a2 Explain the influence of an exponent on the location of a decimal point in each number.

Strand: Number Operations (Real Numbers)

Family: Understanding base Ten Number System

Progress Indicator: *H.NO.1a Using exponents and scientific notation to represent quantities and expressions*

Concrete Understandings:
- Understand that positive exponents result in large whole numbers.
- Understand that negative exponents result in representations less than a whole.
- Understand that the exponent number equals the number of places to the right of left of the decimal. See chart below.

Representation:
- Count out the place value of the exponent in writing in scientific notation.
- Understand the following concepts, symbols, and vocabulary: scientific notation, base number, exponent.

Suggested Instructional Strategies:
- Explicit instruction on the rules (e.g., Exponents tell you how many places to move to the right or left of the decimal)
- Model-Lead-Test*

Supports and Scaffolds:
- Graphic organizer
- Reference table (see below)

Common name	Decimal form	Power of 10	Scientific Notation
One millionth	0.000001	10^{-6}	1×10^{-6}
One thousandth	0.001	10^{-3}	1×10^{-3}
One hundredth	0.01	10^{-2}	1×10^{-2}
One tenth	0.1	10^{-1}	1×10^{-1}
One	1	10^{0}	1×10^{0}
Ten	10	10^{1}	1×10^{1}
One hundred	100	10^{2}	1×10^{2}

One thousand	1,000	10^3	1×10^3
One million	1,000,000	10^6	1×10^6
One billion	1,000,000,000	10^9	1×10^9
One trillion	1,000,000,000,000	10^{12}	1×10^{12}

CCSS: K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

CCC: K.NO.1b2 Identify the set that has more.

Strand: Number Operations (Real Numbers)

Family: Determine Relative Position of Whole Numbers

Progress Indicator: *E.NO.1b Developing an understanding of number and principles of quantity (e.g., hold up 5 fingers at once to show 5, locate things in 2s without counting; using number words to indicate small exact numbers or relative change in quantity - more, small)*

Concrete Understandings:
- Understand the meaning of more, less or equal (the same).
- Understand that as counting increases, the quantities represented increases.
- Count objects up to 5.

Representation:
- Students can distinguish which quantity represents more than others.
- Understand the following concepts, symbols, and vocabulary: more.

Suggested Instructional Strategies:
- Example, non-example training (e.g., this is greater, this is greater, this is greater, this is not greater) *
- Task analysis
    - Count each set (include a strategy for keeping up with what has been counted).
    - Compare the number of items in each set using a number line.
    - State/record the higher number.
- Teach in a real-world context

Supports and Scaffolds:
- Manipulatives to place in sets to compare numbers
- Interactive whiteboards
- Assistive Technology
- Template or jig to place manipulatives in
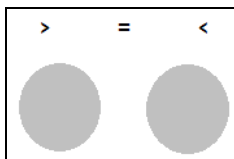
- Number line with numbers written in increasing size (gradually fade this support away so students eventually use regular number line)
- Regular number line
- Hundreds Chart

CCSS: 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
  a) Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
  b) Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

CCC: 6.NO.1d2 Locate positive and negative numbers on a number line.

Strand: Number Operations (Real Numbers)

Family: Determine Relative Position of Whole Number

Progress Indicator: *M.NO.1d Representing integers (positive/negative numbers) and locating them on a number line*

Concrete Understandings:
  - Match to same on number line.
  - Recognize how values of numbers represent the distance of that number from zero.
  - Recognize that on a number line all the numbers to the right of zero are positive and all the numbers to the left of zero are negative.

Representation:
  - Recognize that negative numbers have a negative symbol (-) before the number.
  - Recognize that positive numbers either have a (+) symbol or no symbol before the number.
  - Understand the meaning of "0" and where it falls on a number line.

Suggested Instructional Strategies:
  - Multiple exemplars training*
  - Time delay*
  - Apply to real world activities when possible.
    o Put positive and negative numbers in the context of a boat on the ocean (the boat at 0 on a number line) with everything above being positive and everything below being negative. See video resource for further explanation
    o <u>Video resource</u>

Supports and Scaffolds:
  - Colored number line (e.g., red numbers for negative, green numbers for positive)
  - Raised or textured number line

- Interactive whiteboards or other technology
- Thermometer for weather related activities
- <u>Video support</u>

CCSS: 6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

CCC: 6.NO.1d4 Select the appropriate meaning of a negative number in a real-world situation.

Strand: Number Operations (Real Numbers)

Family: Determine Relative Position of Whole Numbers

Progress Indicator: *M.NO.1d Representing integers (positive/negative numbers) and locating them on a number line*

Concrete Understandings:
- Describe negative numbers as numbers less than zero.
- Understand less/same/more in context (e.g., temperature, ground level).

Representation:
Select pictorial representations of less than 0 in a real-world context.

Suggested Instructional Strategies:
- Teach using real-world context: Use a poster and paint a tree with roots, a thermometer, or a boat on the ocean, with water and fish below. Use stories to talk about above and below ground/water level or freezing and the meaning of a negative value. Then introduce a number line and teach beginning with zero, numbers increase in value to the right and decrease in value to the left, becoming negative when to the left of zero.
  - Other real-world activities: credit and debit examples, temperature, calories, supply/demand
- Model the thinking within context (e.g., tug of war- could do it physically or on computer with number line, temperature, above/below sea level)

Supports and Scaffolds:
- Visual template with vertical or horizontal number lines on the picture (e.g., building a pool. Dig a 6-foot hole: ground level would be 0, bottom of hole -6)
- Interactive whiteboards or other technology to manipulate representations

CCSS: 2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

CCC: 2.NO.2a19 Combine up to 3 sets of 20 or less.

Strand: Number and Operations; (Whole numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *E.NO.2a Representing addition and subtraction in multiple ways composing/ decomposing numbers, diagrams, using objects, arrays, equations, number lines), including regrouping*

Concrete Understandings:
- Given a number, make a set of objects within 20.
- Combine sets to get a total.

Representation:
- Given a picture of base 10 blocks representing up to 3 sets, find the sum.
- Understand the following concepts, symbols, and vocabulary: add.

Suggested Instructional Strategies:
- Task analysis
- Teach using manipulatives

Supports and Scaffolds:
- Place value template
- Number line
- Calculator
- Three colors of counters
- Double 10 frame
    - See website for further explanation

CCSS: 1.OA.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

CCC: 2.NO.2b1 Use commutative properties to solve addition problems with sums up to 20 (e.g., 3+8=11 therefore 8+3=__).

Strand: Number and Operations; (Whole numbers).

Family: Perform Operations with Whole Numbers

Progress Indicator:*E.NO.2b Explaining or modeling the relationship between addition and subtraction*

Concrete Understandings:
- Count two sets of objects.
- Count two provided sets of cubes that represent the commutative property (3 green cubes and 5 blue cubes equals 8 cubes; 5 green cubes and 3 blue cubes also equals 8).

Representation:
- Identify the corresponding equation that illustrates the commutative property (e.g., when given 2+8, can select 8+2 as the equation that illustrates the commutative property).
- Understand the following concepts, symbols, and vocabulary: add, commutative property (when two numbers are added, the order of the addends does not change the sum), order

Suggested Instructional Strategies:
- Use a number balance to investigate the commutative property (e.g., if 8 and 2 on one side of the balance equals 10, then if I put a weight on 2 first and then on 8, it will also be 10).
- Video resource
- Model using the commutative properties to solve addition problems

Supports and Scaffolds:
- The following website provides a tool for practicing balancing equations
- Calculator
- Interactive whiteboard
- Manipulatives

CCSS: 1.OA.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

CCC: 2.NO.2b2 Use associative property to solve addition problems with sums up to 20.

Strand: Number and Operations; (Whole numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *E.NO.2b Explaining or modeling the relationship between addition and subtraction*

Concrete Understandings:
- Add sets of objects within 10.
- Count a set of objects within 20.
- Combine 2 sets that total up to 20 objects.

Representation:
- Identify the corresponding equation that illustrates the associative property (e.g., when given (3+2) +8, can select 3+(2+8) as the equation that illustrates the associative property).
- Understand the following concepts, symbols, and vocabulary: add, associative property (when three or more numbers are added, the grouping of the addends does not change the sum), order.

Suggested Instructional Strategies:
- Video resources
- Model using the associative property

Supports and Scaffolds:
- Calculator
- Interactive whiteboard
- Manipulatives

CCSS: 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

CCC: 3.NO.2b1 Use the relationships between addition and subtraction to solve problems.

Strand: Number and Operations; (Whole numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *E.NO.2b Explaining or modeling the relationship between addition and subtraction*

Concrete Understandings:
- Identify situations where you would add or subtract numbers.
- Use base ten blocks to correctly solve addition/subtraction problems.

Representation:
- Use base ten blocks to add and subtract numbers within 999.
- Understand concepts, symbols, and vocabulary for: add, subtract, sum, difference, total.

Suggested Instructional Strategies:
Use a problem-solving template with base 10 blocks; have students build the start number in one of the spaces then either add another pile or subtract by removing base 10 blocks from the start pile

Supports and Scaffolds:
- Calculator
- Manipulatives

CCSS: 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

CCC: 3.NO.2d3 Solve multiplication problems with neither number greater than 5.

Strand: Number Operations (Real Numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *E.NO.2d Modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers*

Concrete Understandings:
- Create an array of sets (e.g., 3 rows of 2 objects) from a set of objects.
- Use graph paper or draw an array that has up to 5 columns and up to 5 rows.
- Count a set of objects within 25.

Representation:
Identify or draw pictorial representation of an array that matches the multiplication problem.

Suggested Instructional Strategies:
- Task analysis (e.g., state the problem (2 sets of 3), draw out the array for the problem, count the total, enter the answer)
- Mnemonics or memory aids
    - o Use familiar songs or raps and replace the words with multiplication facts.
    - o Use kinesthetic activities such as dancing or marching. Students say multiplication facts as they move.
    - o Times Tales: a mnemonic program that associates silly stories with multiplication facts
- Counting strategies (i.e., repeated addition with whole numbers)
- Teach multiplication using concrete objects.
- Short drill sessions using a multiplication table

Supports and Scaffolds:
- Calculator
- Raised grid (to keep structure of array) or graph paper
- Interactive whiteboards or other technology to manipulate representations
- Large posters of math tables to hang on classroom walls
- Assistive Technology

CCSS: 4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

CCC: 4.NO.2f1 Identify multiples for a whole number (e.g., 2= 2, 4, 6, 8, 10).

Strand: Number and Operations; (Whole numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *E.NO.2f Identifying factors and multiples of numbers*

Concrete Understandings:
Add a one-digit number to a two-digit number using manipulatives or other strategies.

Representation:
Identify multiples of whole numbers using a 100s chart with markers.

Suggested Instructional Strategies:
- Use calculators to explore the patterns of multiples when skip counting by a given number.
- Mnemonics or memory aids
  - Use familiar songs or raps and replace the words with multiplication facts.
  - Use kinesthetic activities such as dancing or marching. Students say multiplication facts as they move.
  - Times Tales: a mnemonic program that associates silly stories with multiplication facts
- Counting strategies (i.e., repeated addition with whole numbers)
- Teach multiples using concrete objects
- Short drill sessions using a multiple

Supports and Scaffolds:
- 100s chart with markers or counters to mark multiples
- Interactive whiteboards or other technology to manipulate representations
- Large posters of math tables to hang on classroom walls
- Assistive Technology
- Number line

CCSS: 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

    a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

    b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then –(*p*/*q*) = (–*p*)/*q* = *p*/(–*q*). Interpret quotients of rational numbers by describing real-world contexts.

    c) Apply properties of operations as strategies to multiply and divide rational numbers.

    d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

CCC: 7.NO.2i1 Solve multiplication problems with positive/negative numbers.
Strand: Number Operations (Real Numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *M.NO.2i Using operations with rational numbers; representing rational numbers and approximations of irrational numbers on a number line*

Concrete Understandings:
- Create an array of objects into groups to model the role of equal groups in a multiplication situation.
- Create an array of objects for the mathematical equation and match answer symbol (+ or -) following multiplication rules for an equation.

Representation:
- Create pictorial array for the mathematical equation and match answer symbol (+ or -) following multiplication rules for an equation.
- Understand the following concepts, symbols, and vocabulary: positive number, negative number.

Suggested Instructional Strategies:
- Explicit rules for multiplying positive and negative numbers (i.e., signs are same, product is positive; signs are different, product is negative)
- Explicit instruction on multiplication
- Task analysis (e.g., steps to solve multiplication problem and then add steps to review signs, apply rule, and select answer)

Supports and Scaffolds:
- Use number line
- Use calculator

- Cheat sheet of rules
- Graphic organizer
- Assistive Technology
- Manipulatives
- Interactive white board technology

CCSS: 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

  a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

  b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

  c) Apply properties of operations as strategies to multiply and divide rational numbers.

  d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

CCC: 7.NO.2i2 Solve division problems with positive/negative numbers.

Strand: Number Operations (Real Numbers)

Family: Perform Operations with Whole Numbers

Concrete Understandings:
Create an array of objects for the mathematical equation and match answer symbol (+ or -) following division rules for an equation.

Representation:
  • Create pictorial array for the mathematical equation and match answer symbol (+ or -) following division rules for an equation.
  • Understand the following concepts, symbols, and vocabulary: positive number, negative number.

Suggested Instructional Strategies:
  • Explicit teaching of rules (i.e., signs are same, quotient is positive; signs are different, quotient is negative)
  • Represent division as "undoing" multiplication to demonstrate a negative divided by a positive is a negative.
  • Explicit instruction in division

Supports and Scaffolds:
  • Use number line
  • Use calculator
  • Provide "cheat sheet" of rules

CCSS: N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.SSE.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈ 1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

CCC: H.NO.1a1 Simplify expressions that include exponents.

Strand: Number Operations (Real Numbers)

Family: Perform Operations with Whole Numbers

Progress Indicator: *H.NO.1a Using exponents and scientific notation to represent quantities and expressions*

Concrete Understandings:
- Identify expressions with exponents.
- Create a model with objects to show that the exponent of a number says how many times to use the number in a multiplication. (Substitute a chip for each "a.")
- $a^7 = a \times a \times a \times a \times a \times a \times a =$ aaaaaaa

Representation:
- Simplify expression into expanded form $(x^4)(x^3)$= (xxxx)(xxx)
- Simplify expression into the simplest form $(x^4)(x^3)$=(xxxx)(xxx) =(xxxxxxx)=x^7
- Understand the following concepts, symbols, and vocabulary: expression, exponent, raising to a power.

Suggested Instructional Strategies:
- Explicitly teach rules for simplification.
- Multiple exemplars (example/non-example) expression with exponents*

Supports and Scaffolds:
- Templates
- Calculator

CCSS: 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

CCC: 3.NO.2e1 Solve or solve and check one or two step word problems requiring addition, subtraction or multiplication with answers up to 100.

Strand: Number Operations (Real Numbers)

Progress Indicator: *E.NO.2e Describing relationships between addition-multiplication; multiplication-division; addition-subtraction; why commutative property does not apply to subtraction or division*

Concrete Understandings:
- Combine (+), decompose (-), and multiply (x) with concrete objects; use counting to get the answers.
- Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away; the difference) in a word problem.

Representation:
- Draw or use a representation of the word problem.
- Add on or count back depending upon the words in the problem.
- Understand the concepts, symbols, and vocabulary for: +, =, -, x.

Suggested Instructional Strategies:
- Task analysis for each type of problem
- Use counting strategies
- Use number patterns (i.e., skip counting)
- Modeling problem solving with supports
- Explicit strategies for self-check of answers
- Explicit teaching of regrouping to solve addition and subtraction problems

Supports and Scaffolds:
- Use calculator
- Use template/graphic organizer to fill in steps of word problems
- Raised grid (to keep structure of array) or graph paper for multiplication or addition problems
- Interactive whiteboards or other technology to manipulate representations
- Provide manipulatives or picture representations with symbols included
- Highlight text that provides important information/vocabulary

CCSS: 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which several shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 ×? = 48, 5 = _ ÷ 3, 6 × 6 = ?.

CCC: 4.NO.2d7 Determine how many objects go into each group when given the total number of objects and groups where the number in each group or number of groups is not > 10.

Strand: Number Operations (Real Numbers)

Progress Indicator: *E.NO.2d Modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers*

Concrete Understandings:
Create an array of objects given a specific number of rows and the total number, place one object in each group/row at a time.

Representation:
- Draw an array using the given information.
- Understand the following concepts, symbols, and vocabulary for: ÷, =.

Suggested Instructional Strategies:
- Teach division as the inverse of multiplication
- Explicit teaching of steps for division
- Task analysis (e.g., identify the number of groups, put one object in each group for total number of objects, count one group of objects, write down number, count second group to verify total, write answer)

Supports and Scaffolds:
- Use a template/graphic organizer to create array
- Use a calculator
- Interactive whiteboards or other technology to manipulate representations
- Use manipulatives for context
- Provide structure for each group/row

CCSS: 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*

CCC: 4.NO.2d8 Match an accurate addition and multiplication equation to a representation.

Strand: Number Operations (Real Numbers)

Progress Indicator: *E.NO.2d Modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers*

Concrete Understandings:
- Select the representation of manipulatives on a graphic organizer to show addition/multiplication equation.
- Match to same for representations of equations (may be different objects but same configuration).

Representation:
- Select a representation to place under each numeral in addition equation.
- State what the numbers represent in a multiplication equation (e.g., first number is number of sets, second number is number within each set).
- Select a pictorial representation of an array that matches the multiplication problem.
- Understand the following concepts, symbols, and vocabulary for: +, x, =, factor, sum, total, product.

Suggested Instructional Strategies:
- Task analysis (e.g., state the problem (2 sets of 3), count the objects in the array in each row/set, select the answer for the problem from choices, enter the answer)
- Counting strategies to select the correct answer or eliminate incorrect answers

Supports and Scaffolds:
- Interactive whiteboards or other technology
- Visual representations that can be manipulated
- Graphic organizer or template

CCSS: 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

CCC: 5.NO.2a1 Solve problems or word problems using up to three-digit numbers and addition or subtraction or multiplication.

Strand: Number Operations (Real Numbers)

Family: Modeling/Symbolizing Operations (Problem Solving) with Whole Numbers

Progress Indicator: *M.NO.2a working flexibility with common addition, subtraction, multiplication, and division situations*

Concrete Understandings:
- Combine (+) or decompose (-) with concrete objects; use counting to get the answers.
- Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away, difference) in a word problem.

Representation:
- Draw or use a representation of the word problem.
- Understand symbols +, =, -,
- Add on or count back depending upon the words in the problem.
- Translate wording into numeric equation.

Suggested Instructional Strategies:
- Task analysis for each type of problem
- Use counting strategies
- Use number patterns (i.e., skip counting)
- Modeling problem solving identifying key words
- Explicit teaching of regrouping
- Explicit teaching of carrying to the next place value
- Explicit teaching of regrouping to solve addition and subtraction problems

Supports and Scaffolds:
- Addition and subtraction template to fill in the steps of the word problem (___ + ____ = ____; a horizontal structure with boxes above the first number for regrouping)
- Use a calculator
- Interactive whiteboards or other technology to manipulate representations
- Provide meaningful manipulatives or picture representations with symbols included
- Highlight text that provides important information/vocabulary

CCSS: 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models

CCC: 5.NO.2a5 Solve word problems that require multiplication or division.

Strand: Number Operations (Real Numbers)

Family: Modeling/Symbolizing Operations (Problem Solving) with Whole Numbers

Progress Indicator: *M.NO.2a Working flexibility with common addition, subtraction, multiplication, and division situations*

Concrete Understandings:
- Combine (x) or decompose (÷) with concrete objects; use counting to get the answers.
- Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away) in a word problem.
- Understand concept of division: sharing or grouping numbers into equal parts.
- Understand concept of multiplication: the result of making some number of copies of the original.

Representation:
- Draw or use a representation of the word problem.
- Symbols ÷, =, x
- Identify purpose to either find a total (multiplication) or one component (number of sets or number within each set for division), depending upon the problem
- Translate wording into numeric equation.

Suggested Instructional Strategies:
- Task analysis for each type of problem
- Use counting strategies
- Use number patterns (i.e., skip counting)
- Modeling problem solving identifying key words
- Explicit teaching of regrouping to solve addition and subtraction problems
- Explicit teaching of steps of multiplication, division (i.e., divide, multiply, subtract, drop down the next digit)
- Explicit instruction on vocabulary associated with a decision to multiply or divide

Supports and Scaffolds:
- Multiplication or division template to fill in the steps of the word problem (___ x ____ = ____; a horizontal structure with boxes for /regrouping)
- Use a calculator
- Interactive whiteboards or other technology to manipulate representations
- Provide meaningful manipulatives or picture representations with symbols included
- Highlight text that provides important information/vocabulary

- Multiplication and division tables

CCSS: 6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for*

CCC: 5.SE.1b1 Evaluate whether or not both sides of an equation are equal.

Strand: Symbolic Expression

Family: Modeling/Symbolizing Operations (Problem Solving) with Whole Numbers

Progress Indicator: *M.SE.1b Writing, interpreting, and using expressions, equations, and inequalities (including using brackets, parentheses, or braces)*

Concrete Understandings:
- Determine if sets are equal/not equal.
- Model an equation with objects.

Representation:
- Complete operations required in equation.
Understand the concepts, symbols, and vocabulary for: =, ≠, +, -, ÷, ×

Suggested Instructional Strategies:
- Task analysis (solve each side of the equation, compare answers, determine if = or not ≠)
- Use manipulatives and a scale to give concrete examples of equal and not equal.

Supports and Scaffolds:
- Calculator
- Number line
- Written task analysis for independence
- Assistive technology or voice output device

CCSS: 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x = p = q and px = q for cases in which p, q, and x are all non-negative rational numbers

CCC: 6.NO.2a6 Solve problems or word problems using up to three-digit numbers and any of the four operations

Strand: Number Operations (Real Numbers)

Family: Modeling/Symbolizing Operations (Problem Solving) with Whole Numbers

Progress Indicator: *M.NO.2a Working flexibility with common addition, subtraction, multiplication, and division situations*

Concrete Understandings:
- Recognize the intended outcome of a word problem without an operation.
- Combine (+) or decompose (-) with concrete objects; use counting to get the answers.
- Combine (x) or decompose (÷) with concrete objects; use counting to get the answers.
- Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away, difference) in a word problem.
- Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away) in a word problem. Understand that division is sharing or grouping numbers into equal parts and multiplication is the result of making some number of copies of the original.

Representation:
- Draw or use a representation of the word problem.
- Understand symbols +, -, ÷, =, x
- Identifying purpose to either find a total (sum for addition or product for multiplication), remaining amount (difference for subtraction) or one component (number of sets or number within each set-dividend or divisor for division), depending upon the words in the problem.
- Translate wording into numeric equation.

Suggested Instructional Strategies:
- Task analysis for each type of problem
- Use counting strategies
- Use number patterns (i.e., skip counting)
- Modeling problem solving, identifying key words
- Explicit teaching of borrowing and regrouping
- Explicit teaching of carrying to the next place value
- Explicit teaching of steps of multiplication, division, or long division (i.e., divide, multiply, subtract, drop down the next digit)

Supports and Scaffolds:
- Operation template to fill in the steps of the word problem (___ x ____ = ____; (___ + ____ = ____; a horizontal structure with boxes for carrying/regrouping)
- Use a calculator
- Interactive whiteboards or other technology to manipulate representations
- Provide manipulatives or picture representations with symbols included
- Highlight text that provides important information/vocabulary