Elements of the Instructional Families: Data, Probability and Statistics

Teaching Data, Probability, and Statistics

All the CCCs in this document relate to teaching Data, Probability, and Statistics. Below are some additional website resources that may be helpful:

Math Teaching Ideas

Math Concepts

Math Activities and Games

Math Lessons Virtually

Math Courses

Teaching Children Math

CCC Mathematics | Data, Probability and Statistics

CCSS: 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

CCC: 1.DPS.1a2 Select questions that ask about "How many" and represent up to three categories that can be concretely represented.

Strand: Data, Probability and Statistics

Family: Formulate Questions/Plan research

Progress Indicator: *E.DPS.1a Posing questions of interest that can be answered by counting or collecting data (e.g., concrete comparisons about students, classroom materials, science topics) with teacher guidance*

Concrete Understandings:
Match an object or representation to the exact replica.

Representation:
Match a question with a set of data (e.g., match the question about counting bears to the set of counting bears).

Suggested Instructional Strategies:
- Model selecting questions and creating categories.
- Task analysis
  - Select a topic (e.g., ice cream).
  - Select a question (e.g., what is your favorite flavor of ice cream?).
  - Select categories based on your questions (e.g., chocolate, vanilla, strawberry).

Supports and Scaffolds:
- Graphic organizer with steps of task analysis
- Photos representing topics that can be the focus of the research questions (e.g., 1. ice cream, 2. animals in a zoo, 3. movies)
- Photos representing categories for topics (e.g., 1. chocolate, vanilla, strawberry ice cream; 2. lions, monkeys, elephants; 3. Shrek, Spiderman, Journey to the Center of the Earth)

CCSS: 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

CCC: 1.DPS.1c1 Using a picture graph, represent each object/person counted on the graph (1:1 correspondence) for 2 or more categories.

Strand: Data, Probability and Statistics

Family: Represent and Interpret Data

Progress Indicator: *E.DPS.1c Collecting and organizing/representing data (e.g., picture graphs, tally charts, bar graphs)*

Concrete Understandings:
- Understand how data is organized on a picture graph.
- Can match to correct category (match to same)

Representation:
Understand that each person can only represent one piece of data.

Suggested Instructional Strategies:
- Task analysis
    - Use established categories (e.g., flavors of ice cream from CCC 1.DPS.1a2).
    - Provide 10 students with a means of choosing their favorite flavor (e.g., a sheet with the 3 choices for students to circle; 3 separate choice cards (each card with a different choice).
    - Have student who is collecting data sit at a table with the picture graph.
    - Each of the 10 students comes to the table one at a time and provides their choice.
    - As each student communicates their choice (e.g., chocolate, vanilla, or strawberry) the student matches the symbol for the flavor chosen with the symbol on the graph.
    - The student collecting the data places the symbol in the matching column, building the column higher each time that choice (e.g., vanilla) is made.
    - Note: If students are given 3 choice cards, they can simply give their choice to the student collecting data and the student can use the card to represent the choice on the graph.
- Task analysis involving counting
    - The students are provided with a table representing the categories.

What is the favorite flavor of ice cream?	
Chocolate	
Vanilla	
Strawberry	

    - The teacher asks the students to raise hand or indicate if their favorite flavor is chocolate.

- Students indicate and the teacher tallies votes on white board.
- Teacher repeats process for next two flavors.
- After data is collected the tally marks for each category are counted by teacher and students together.
- Students write the total votes into their table.

What is the favorite flavor of ice cream?	
Chocolate	5
Vanilla	5
Strawberry	2

- Students count out the number of symbols for first category and place vertically on picture graph.
- Repeat with remaining categories.

Supports and Scaffolds:
- Assistive technology (e.g., voice output)
- Interactive whiteboard
- Alternate materials for making choices between categories (e.g., 3 flavors of ice cream to taste, recorded sounds of the three zoo animals, short movie clips from movie choices)
- Picture graph with raised dividers between the columns representing different categories and/or raised cells that represent each student's choice
- Object representations to use on graph as opposed to picture representations

CCSS: 2.MD.10 Draw a picture graph and a bar graph to represent a data set with up to four categories. Solve simply put together, take-apart, and compare problems using information presented in a bar graph.

CCC: 2.DPS.1c2 Organize data by representing categorical data on a pictorial graph or bar graph.

Strand: Data, Probability and Statistics

Family: Represent and Interpret Data

Progress Indicator: *E.DPS.1c Collecting and organizing/representing data (e.g., picture graphs, tally charts, bar graphs)*

Concrete Understandings:
- Identify a picture or bar graph.
- Identify a data set represented with pictorial representations (e.g., pictures of apples, pictures of oranges).

Representation:
- Know parts of a picture or bar graph.
- Identify a data set represented with numbers.

Suggested Instructional Strategies:
- Task analysis
    - Use established categories (e.g., flavors of ice cream from CCC 1.DPS.1a2).
    - Provide 10 students with a means of choosing their favorite flavor (e.g., a sheet with the 3 choices for students to circle; 3 separate choice cards (each card with a different choice).
    - Have student who is collecting data sit at a table with the picture graph.
    - Each of the 10 students comes to the table one at a time and provides their choice.
    - As each student communicates their choice (e.g., chocolate, vanilla, or strawberry) the student matches the symbol for the flavor chosen with the symbol on the graph.
    - The student collecting the data places the symbol in the matching column, building the column higher each time that choice (e.g., vanilla) is made.
    - Note: If students are given 3 choice cards, they can simply give their choice to the student collecting data and the student can use the card to represent the choice on the graph.
- Task analysis involving counting
    - The students are provided with a table representing the categories

What is the favorite flavor of ice cream?	
Chocolate	
Vanilla	

Strawberry	

- o The teacher asks the students to raise hand or indicate if their favorite flavor is chocolate.
- o Students indicate and teacher tallies votes on white board.
- o Teacher repeats process for next two flavors.
- o After data is collected the tally marks for each category are counted by teacher and students together.
- o Students write the total votes into their table.

What is the favorite flavor of ice cream?	
Chocolate	5
Vanilla	5
Strawberry	2

- o Students count out the number of symbols for first category and place vertically on picture graph.
- o Repeat with remaining categories.

Supports and Scaffolds:
- Assistive technology (e.g., voice output)
- Interactive whiteboard
- Alternate materials for making choices between categories (e.g., 3 flavors of ice cream to taste, recorded sounds of the three zoo animals, short movie clips from movie choices)
- Picture graph with raised dividers between the columns representing different categories and/or raised cells that represent each student's choice
- Object representations to use on graph as opposed to picture representations

CCSS: 2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simply put-together, take apart, and compare problems using information presented in a bar graph.

CCC: 2.DPS.1e2 Compare the information shown in a bar graph or picture graph with up to four categories. Solve simple comparisons of how many more or how many less.

Strand: Data, Probability and Statistics

Family: Draw Conclusions from Data Collection

Progress Indicator: *E.DPS.1e Describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs)*

Concrete Understandings:
- Identify the categories in a graph.
- Understand the concept of more and less.
- Count sets within a category
- Understand how data is organized on a picture graph.
- Can match to correct category (match to same)

Representation:
- Understand that each person can only represent on piece of data.
- Identify data set with some number (e.g., bar graph representing 5 pencils).
- Identify a picture or bar graph.

Suggested Instructional Strategies:
- Teach the concept of more or less using example, non-example; apply to data on graph.
- Use or create a graph that provides a visual of the values in each category such as a bar graph.
- Teach the concept of more or less using a number line.
- Task analysis
    - Select two categories for graph for comparison.
    - Count or identify the total for the first category.
    - Mark the total on a graphic organizer with a number line (see below for example).
    - Repeat for second category.
    - Identify which category has more OR which category has less.
    - Based on the category that you identified in the previous step, (e.g., the category that has more), count or back to the remaining category by jumping the numbers.
    - Record the number of jumps.

Supports and Scaffolds:
- Number line
- Concrete objects for graph
- Graphic organizer with number line and question to be answered

Equation prompt

First fact	Sign	Second fact	Sign	Last fact
	+ −		**=**	

1   2   3   4   5   6   7   8   9   10

Add ⟶   ⟵ Subtract

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CCSS: 3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one-and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

CCC: 3.DPS.1g1 Collect data and organize it into picture or bar graph.

Strand: Data, Probability and Statistics

Family: Represent and Interpret Data

Progress Indicator: *E.DPS.1g Collecting data and representing data (e.g., bar graphs, frequency tables, line plots)*

Concrete Understandings:
- Identify data set based on a single attribute (e.g., pencils vs. markers).
- Identify data set with (e.g., this bar represents a set with more).
- Organize the data into a picture or bar graph using objects that represent one piece of data (may have number symbols).
- Properly label graph (e.g., axes on bar graph).

Representation:
- Identify data set with some number (e.g., bar graph representing 5 pencils).
- Identify a picture and bar graph.

Suggested Instructional Strategies:
- Task analysis (e.g., Decide on your question, create categories, decide on source of data, collect data based on categories, count data sets, organize data, create graph)
- Match to same
- Graphic organizer to establish sets

Supports and Scaffolds:
- Template (e.g., graphic organizer) or structure for collecting and organizing data
- Velcro numbers on a graph
- Interactive whiteboard
- Computer Software
- Grid paper to construct graph
- Pre-made graphs
- Assistive technology/voice output device
- Self-monitoring task analysis for student independence

CCSS: 3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one-and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

CCC: 4.DPS.1g3 Collect data and organize it into a graph (e.g. picture graph, line plot, bar graph).

Strand: Data, Probability and Statistics

Family: Represent and Interpret Data

Progress Indicator: *E.DPS.1g Collecting data and representing data (e.g., bar graphs, frequency tables, line plots)*

Concrete Understandings:
- Identify data set based on a single attribute (e.g., pencils vs. markers).
- Identify data set with more of less (e.g., this bar represents a set with more).
- Organize the data into a graph using objects where each object represents one piece of data (may have number symbols).
- Properly label graph (e.g., axes on bar graph).

Representation:
- Identify components of graphs (picture, line, plot, bar).
- Identify data set with some number (e.g., bar graph with 5 pencils).
- Organize the data into a pictorial graph.

Suggested Instructional Strategies:
- Task analysis for categorical data (e.g., Decide on your question, create categories, decide on source of data, collect data based on categories, count data sets, organize data, select best graph for data, create graph)
- Task analysis for continuous data (e.g., Decide you topic for data collection, collect data, identify the least and greatest numbers within the data set, create line graph using the least to greatest range, plot each number on the line graph)
- Match to same (for categorical or continuous data)
- Graphic organizer to establish sets
- Use number lines to teach use of a line plot
- Discussion (e.g., whole class think aloud after a teach model, think pair share)

Supports and Scaffolds:
- Template (e.g., graphic organizer) or structure for collecting and organizing data
- Velcro numbers on a graph
- Interactive whiteboard
- Computer software
- Grid paper to construct graph
- Pre-made graphs

- Assistive technology/voice output device
- Self-monitoring task analysis for student independence

CCSS: 6.SP.5 Summarize numerical data sets in relations to their context such as by: Giving quantitative measures of center (median and/or mean) and variability as well as describing any overall pattern and striking deviations from the overall pattern with reference to the context in which the data were gathered.

CCC: 6.DPS.1d3 Select statement that matches mean, mode, and spread of data for 1 measure of central tendency for given data set.

Strand: Data, Probability, and Statistics

Family: Represent and Interpret Data

Progress Indicator: *M.DPS.1d Identifying the range, three common measures of central tendency (mean, median, and mode) and interpreting the mean as a fair share and a center of balance*

Concrete Understandings:
- Identify the lowest to highest value in a data set given a number line and matching symbols.
- Arrange data from lowest to highest.
- Identify the median.
- Identify the representation (plastic snap cubes, wiki sticks) of the mode.
- Use concrete materials to produce the mean (leveled plastic snap cubes).

Representation:
- Identify the mode and the spread of the data using a line drawing of the distribution.
- Calculate the mean using pre-slugged template of data points.
- Understand the following concepts and vocabulary: mean, mode, and spread of data.

Suggested Instructional Strategies:
- Task analysis using template (e.g., steps to find the mean)
- Task analysis for the spread of the data
- Explicit teaching of the definitions of mean, mode, and spread of data
- Leveling the bars to find the mean (e.g., given 6 groups of various numbers of cubes, students level the bars into 6 equal groups. The number within each group is the mean.)

Supports and Scaffolds:
- Calculator
- Graphic organizer for mode
- Raised line around the distribution of the data (e.g., wiki sticks)
- Template for finding mean
- Pre-slugged template
- Interactive whiteboard
- Computer software
- Self-monitoring task analysis for student independence

CCSS: 7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about the two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

CCC: 7.DPS.1k1 Analyze graphs to determine or select appropriate comparative inferences about two samples or populations.

Strand: Data, Probability and Statistics

Family: Draw Conclusions from Data Collection

Progress Indicator: *M.DPS.1k Supporting claims about the results of investigations (e.g., coordinating among the measures of central tendency and variability)*

Concrete Understandings:
- Understand basic information from simple graphs (e.g., interpret a bar graph using the understanding that the taller column on a graph has a higher frequency, the shorter column on a graph has a lower frequency).
- Identify a representation of two bar graphs (one category apiece) as having greater or less frequency of members/events related to a single variable (e.g., *Example, compare number of boys in soccer to girls in two graphs).*
- Make a comparison between two graphs.

Representation:
- Identify a pictorial representation of two bar graphs (one category apiece) as having greater or less frequency of members/events related to a single variable (e.g.*, compare number of boys in soccer to girls in two graphs).*
- Understand the concept, symbols and vocabulary for: more, less, higher, lower.

Suggested Instructional Strategies:
- Task analysis to analyze graph (e.g., look at 1st graph, identify the column of interest, mark, look at 2nd graph
- Identify the column of interest, mark, determine the facts that the graph shoes)
- Model-Lead-Test to demonstrate making inferences*
- Group discussion forum, with teacher model, to create arguments based on graphs
- Include discussions that model the analysis of two graphs and encourage student to provide evidence for the inference (e.g., note minimum and maximum values (range), use measures of central tendency, note any key points and relationships in the graph or data set)

Supports and Scaffolds:
- Raised line around the distribution of the data (e.g., wiki sticks)
- Assistive technology/voice output devices
- Interactive whiteboard
- Highlighters
- Graphs that can be manipulated to make comparisons

- Self-monitoring task analysis for student independence
- Templates with sentence starters

CCSS: 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

CCC: 8.DPS.1h1 Graph bivariate data using scatter plots and identify possible associations between the variables.

Strand: Data, Probability and Statistics

Family: Represent and Interpret Data

Progress Indicator: *M.DPS.1h Displaying data in scatter plots and investigating the association between the variables*

Concrete Understandings:
- Locate points on the x-axis and y-axis an adapted grid (not necessarily numeric).
- Identify a similar distribution when given a choice of three (e.g., when shown a normal distribution, can select a second example of a normal distribution from three choices).

Representation:
- Graph a series of data points on a coordinate grid.
- Identify the associations between the variables using supports.
- Understand the following concepts and vocabulary: best fit line, variable, outliers.

Suggested Instructional Strategies:
- Task analysis for graphing bivariate data
  - Student adds points to data table (number of ice cream cones sold compared to outside temperature).
  - The number of ice cream cones is indicated along the y-axis; the temperature is indicated along the x-axis.
  - The student moves red marker on the y-axis to represent a value from the table. "Place the marker on the y-axis for this number of ice cream cones sold."
  - The student moves green marker on the x-axis to represent a value from the table. "Place the marker on the x-axis for this temperature."
  - Using straws on the x- and y-axes, the student finds the coordinate on the graph represented by the data.
  - The student continues to plot more points (at least three points, not necessarily a perfect relationship).
  - Ask student to place string/straw/yarn along the points.
  - Indicate the direction of the straw. Have the student look at the graph he/she made and ask: "How does the temperature relate to the number of ice cream cones sold?"
  - Student describes the relationship between the two variables. "Warmer weather leads to more ice cream cone sales."
- Explicitly teach three potential outcomes (i.e., as one variable increases the other decreases; as one decreases the other increases; there is no trend).

- Multiple exemplars of the three outcomes*
- System of least prompts to graph data*

Supports and Scaffolds:
- Color coded grid (e.g., uses colors rather numbers)
- Raised grid
- Graphing calculator
- Use manipulatives to show relationships (e.g., transparencies that highlight relationships, straight line object such as spaghetti to find best fit line).
- Self-monitoring task analysis for student independence
- Templates with sentence starters
- Interactive whiteboard
- Assistive technology

CCSS: 8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for tows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

CCC: 8.DPS.1k2 Analyze displays of bivariate data to develop or select appropriate claims about those data

Strand: Data, Probability and Statistics

Family: Draw Conclusions from Data Collection

Progress Indicator: *M.DPS.1k Supporting claims about the results of investigations (e.g., coordinating among the measures of central tendency and variability)*

Concrete Understandings:
- Identify a similar distribution when given a choice of three (e.g., when shown a normal distribution, the student can select a second example of a normal distribution from three choices).
- Identify the appropriate statement when given a relationship between two variables (may use graphic supports such as highlighted transparency of an association).

Representation:
- Explain the associations between the variables using supports (e.g., the selection of the highlighted transparency and make a statement).
- Understand the following concepts and vocabulary: variable, claim

Suggested Instructional Strategies:
- Model-Lead-Test using different associations*
- Guiding questions (e.g., How close is the fit? How sure can you be?)
    - Look at your data.
    - Place the transparency (see below) over your data.
    - Does the highlighting within the transparency cover your data?

Supports and Scaffolds:
- Use manipulatives to show relationships (e.g., transparencies that highlight relationships, straight line object such as spaghetti to find best fit line).
- Interactive whiteboard
- Highlighted scatter plots
- Assistive technology/voice output devices
- Templates with sentence starters

CCSS: S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

CCC: H.DPS.1b1 Complete a graph given the data, using dot plots, histograms, or box plots

Strand: Data, Probability, and Statistics

Family: Represent and Interpret Data

Progress Indicator: *H.DPS.1b Representing data with plots on the real number line (dot plots, histograms, box plots)*

Concrete Understandings:
- Match the source of the values at the bottom of the x-axis with the appropriate category of the related data table.
- Describe the elements within a graph (e.g., in a box plot, the line is the median, the line extending from each box is the lower and upper extreme, and the box shows the lower quartile and the upper quartile).

Representation:
- Complete the steps of the task analysis to complete a box plot.
- Understand the following concepts and vocabulary: quartile, median, intervals, upper and lower extremes, box plot, histograms, dot plots.

Suggested Instructional Strategies:
- Follow steps of task analysis to complete box plot, dot plots, or histograms (these can be found on internet or many calculators).
- Model-Lead-Test*

Supports and Scaffolds:
- Technology (e.g., computers)
- Graphing calculators
- Self-monitoring task analysis for student independence

CCSS: S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data. Recognize possible associations and trends in the data.

CCC: H.DPS.1c1 Use descriptive stats including range, median, mode, mean, outliers/ gaps to describe data set.

Strand: Data, Probability, and Statistics

Family: Represent and Interpret Data

Progress Indicator: *H.DPS.1c Analyzing and summarizing the data resulting from studies using statistical measures appropriate to shape of the data (median, mean) and spread (interquartile range, standard deviation), and using data to support inferences (population parameters, sample size) or explain possible outliers*

Concrete Understandings:
- Given a scatter plot, identify outliers in the data set.
- Identify the highest and lowest value in a data set given a number line and matching symbols (concept of range).
- Identify the representation (use plastic snap cubes to represent the tally showing the number of occurrences) of the concept of mode.
- Identify the concept of median using concrete representations of data (create a bar graph with an odd number of bars using snap cubes, arrange from shortest to tallest, have student place fingers on two outside towers, knock towers over and move inward until they reach the one middle tower left standing).
- Find the mean using concrete materials.

Representation:
- Identify the mode and the spread of the data using a line drawing of the distribution.
- Calculate the mean using pre-slugged template of data points.
- Order data set using numeric symbols.\
- Understand the following concepts and vocabulary: median, mode, mean, outliers.

Suggested Instructional Strategies:
- Task analysis for finding range, median, mode, mean
- Explicit vocabulary instruction for outliers
- Multiple exemplars for outliers*
- Model data descriptions
- Use concrete materials to find the mean (leveled plastic snap cubes: using the same bar graph with snap cubes, re-arrange cubes into equal stacks).

Supports and Scaffolds:
- Template for finding mean
- Assistive technology/voice output devices
- Interactive whiteboard
- Provide a graph of the data set
- Templates with sentence starters
- Manipulatives