**Core Content Connectors by Common Core State Standards: Mathematics, Statistics, and Probability**

MSAA Instructional Resource Guide, Revised May 2024 from the NCSC content developed as part of the National Center and State Collaborative under a grant from the US Department of Education.

This view is designed to mirror the Common Core document while also including the CCCs linked to each CCSS.

Mathematics High School—Statistics and Probability Overview
Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represented, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Interpreting Categorical and Quantitative Data—S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

| CCCs linked to S-ID.1 |
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| H.DPS.1b1 Complete a graph given the data, using dot plots, histograms, or box plots. |

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

| CCCs linked to S-ID.2 |
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| H.DPS.1c1 Use descriptive stats: range, median, mode, mean, outliers/gaps to describe the data set. |
| H.DPS.1c2 Compare means, median, and range of 2 sets of data. |

3. Interpret differences in shape, center, and spread in the context of the data sets, account for possible effects of extreme data points (outliers).

| NO CCCs linked to S-ID.3 |
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4. Use the mean and standard deviation of a data set to for it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

| CCCs linked to S-ID.4 |
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| H.DPS.1c1 Use descriptive stats: range, median, mode, mean, outliers/gaps to describe the data set. |

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequences). Recognize associations and trends in the data.

| CCCs linked to S-ID.5 |
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| H.DPS.1a1 Design study using categorical and continuous data, including creating a question, identifying a sample, and planning for data collection. |
| H.DPS.1c1 Use descriptive stats: range, median, mode, mean, outliers/gaps to describe the data set. |

6. Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

| CCCs linked to S-ID.6 |
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| H.DPS.1d1 Represent data on a scatter plot to describe and predict. |
| H.DPS.1d2 Select an appropriate statement that describes the relationship between variables. |

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

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| CCCs linked to S-ID.7<br>H.PRF.1a1 Interpret the rate of change using graphical representations. |

8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

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| NO CCCs linked to S-ID.8 |

9. Distinguish between correlation and causation

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| NO CCCs linked to S-ID.9 |

Making Inferences and Justifying Conclusions—S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

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| CCCs linked to S-IC.1 |
| H.DPS.1c3 Determine what inferences can be made from statistics. |

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

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| NO CCCs linked to S-IC.2 |

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies observational studies; explain how randomization relates to each.

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| NO CCCs linked to S-IC.3 |

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error using simulation models for random sampling.

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| NO CCCs linked to SI-C.4 |

5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

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| NO CCCs linked to SI-C.5 |

6. Evaluate reports based on data.

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| CCCs linked to S-IC.6 |
| H.DPS.1d3 Make or select an appropriate statement(s) about findings. |
| H.DPS.1d4 Apply the results of the data to a real-world solution. |

Conditional Probability and the Rules of Probability—S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subtests of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

| NO CCCs linked to S-CP.1 |
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2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

| NO CCCs linked to S-CP.2 |
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3. Understand the conditional probability of A given B as P (A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

| NO CCCs linked to S-CP.3 |
|---|

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

| CCCs linked to S-CP.4 |
|---|
| H.DSP.2d Select or make an appropriate statement based on a two-way frequency table. |

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

| CCCs linked to S-CP.5 |
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| H.DSP.2e Select or make an appropriate statement based on real world examples of conditional probability. |

Use the rules of probability of A given B as the fraction of B’s outcomes that also belong to A, interpret the answer in terms of the model.

6. Finding the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and interpreting the answer in terms of the model.

| NO CCCs linked to S-CP.6 |
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7. Apply the Addition Rule, P (A or B) = P(A) + P(B) – P (A and B) and interpret the answer in terms of the model.

| NO CCCs linked to S-CP.7 |
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8. Apply the general Multiplication Rule in a uniform probability model, P (A and B) = P(A) P(B|A) = P(B) P(A|B) and interpret the answer in terms of the model.

| NO CCCs linked to S-CP.8 |
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9. Use permutations and combinations to compute probabilities of compound events and solve problems.

| NO CCCs linked to S-CP.9 |
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Use Probability to Make Decisions—S-MD

Calculate expected values and use them to solve problems

1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

| NO CCCs linked to S-MD.1 |
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2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

| NO CCCs linked to S-MD.2 |
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3. Develop a probability distribution or a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five equations of a multiple-choice test where each questions have four choices, and find the expected grade under various grading schemes.

| CCCs linked to S-MD.3 |
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| H.DPS.2c1 Determine the theoretical probability of multistage probability experiments. |
| H.DPS.2c2 Collect data from multistage probability experiments. |
| H.DPS.2c3 Compare actual results of multistage experiment with theoretical probabilities. |

4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

| NO CCCs linked to S-MD.4 |
|---|

Use probability to evaluate outcomes of decisions

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
    a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
    b. Evaluate and compare strategies based on expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

| NO CCCs linked to S-MD.5 |
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6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

| NO CCCs linked to S-MD.6 |
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7. Analyze decision and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

| CCCs linked to S-MD.7 |
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H.DSP.2b Identify and describe the degree to which something is rated “good” or “bad”/desirable or undesirable based on numerical information.

## Core Content Connectors by Common Core State Standards: Mathematics, Geometry

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Mathematics High School—Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Congruence—G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

| NO CCCs linked to G-CO.1 |
|---|

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

| NO CCCs linked to G-CO.2 |
|---|

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

| CCCs linked to G-CO.3 |
|---|
| H.GM.1c1 Construct, draw or recognize a figure after its rotation, reflection, or translation. |

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

| NO CCCs linked to G-CO.4 |
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5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

| CCCs linked to G-CO.5 |
|---|
| H.GM.1c1 Construct, draw or recognize a figure after its rotation, reflection, or translation. |

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

| NO CCCs linked to G-CO.6 |
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7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

| CCCs linked to G-CO.7 |
|---|
| H.GM.1b1 Use definitions to demonstrate congruency and similarity in figures. |

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

| NO CCCs linked to G-CO.8 |
|---|

Prove geometric theorems

9. Prove theorems about lines and angles. Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

| NO CCCs linked to G-CO.9 |
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10. Prove theorems about triangles, theorems include measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

| NO CCCs linked to G-CO.10 |
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11. Prove theorems about parallelograms. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

| NO CCCs linked to G-CO.11 |
|---|

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment, copying an angle, bisecting a segment, bisecting an angle, constructing perpendicular lines, including the perpendicular bisector of a line segment, and constructing a line parallel to a given line through a point not on the line.

| CCCs linked to G-CO.12 |
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| H.GM.1e1 Make formal geometric constructions with a variety of tools and methods. |

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

| NO CCCs linked to G-CO.13 |
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Similarity, Right Triangles, and Trigonometry—G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

| CCCs linked to G-SRT.1 |
|---|
| H.ME.2b1 Determine the dimensions of a figure after dilation. |

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

| CCCs linked to G-SRT.2 |
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| H.ME.2b2 Determine if 2 figures are similar. |
| H.ME.2b3 Describe or select why two figures are or are not similar. |
| H.GM.1b1 Use definitions to demonstrate congruency and similarity in figures. |
| H.GM.1d1 Use the reflections, rotations, or translations in the coordinate plane to solve problems with right angles. |

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

| NO CCCs linked to G-SRT.3 |
|---|

Prove theorems involving similarity

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely, the Pythagorean Theorem proved using triangle similarity.

| NO CCCs linked to G-SRT.4 |
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5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

| NO CCCs linked to G-SRT.5 |
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Define trigonometric ratios and solve problems involving the right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

| NO CCCs linked to G-SRT.6 |
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7. Explain and use the relationship between the sine and cosine of complementary angles.

| NO CCCs linked to G-SRT.7 |
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8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

| NO CCCs linked to G-SRT.8 |
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Apply trigonometry to general triangles

9. Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

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| NO CCCs linked to G-SRT.9 |

10. Prove the Laws of Sines and Cosines and use them to solve problems.

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| NO CCCs linked to G-SRT.10 |

11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

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| NO CCCs linked to G-SRT.11 |

Circles—G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.

| CCCs linked to G-C.1 |
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2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

| NO CCCs linked to G-C.2 |
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3. Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.

| NO CCCs linked to G-C.3 |
|---|

4. Construct a tangent line from a point outside a given circle to the circle.

| NO CCCs linked to G-C.4 |
|---|

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

| CCCs linked to G-C.5 |
|---|
| H.ME.2b4 Apply the formula to the area of a sector (e.g., are of a slice of pie). |

Expressing Geometric Properties with Equations—G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

| NO CCCs linked to G-GPE.1 |
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2. Derive the equation of a parabola given a focus and directrix.

| NO CCCs linked to G-GPE.2 |
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3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

| NO CCCs linked to G-GPE.3 |
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Use coordinates to prove simple geometric theorems algebraically

4. Use coordinators to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

| NO CCCs linked to F-TF.4 |
|---|

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

| NO CCCs linked to G-GPE.5 |
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6. Find the point on a directed line segment between two given points that partitions the segment in each ratio.

| NO CCCs linked to G-GPE.6 |
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7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

| NO CCCs linked to G-GPE.7 |
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Geometric Measurement and Dimension—G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

| NO CCCs linked to G-GMD.1 |
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2. Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

| NO CCCs linked to G-GMD.2 |
|---|

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

| NO CCCs linked to G-GMD.3 |
|---|

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

| NO CCCs linked to G-GMD.4 |
|---|

Modeling with Geometry—G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

| CCCs linked to G-MG.1 |
| --- |
| H.ME.1b1 Describe the relationship between the attributes of a figure and the changes in the area or volume when 1 attribute is changed. |

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

| NO CCCs linked to G-MG.2 |
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3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

| CCCs linked to G-MG.3 |
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| H.ME.2b5 Apply the formula of geometric figures to solve design problems (e.g., designing an object or structure to satisfy physical restraints or minimize cost). |

## Core Content Connectors by Common Core State Standards: Mathematics, Functions

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Mathematics High School—Functions Overview
Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Interpreting Functions—F-IF

Understand the concepts of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x)

| NO CCCs linked to F-IF.1 |
|---|

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

| NO CCCs linked to F-IF.2 |
|---|

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subtest of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f (n + 1) = f (n) + f (n – 1) for n ≥ 1.

| NO CCCs linked to F-IF.3 |
|---|

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

| NO CCCs linked to F-IF.4 |
|---|

5. Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function h (n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

| NO CCCs linked to F-IF.5 |
|---|

6. Calculate and interpret the average rate of change of function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

| NO CCCs linked to F-IF.6 |
|---|

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cubic root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are

NO CCCs linked to F-IF.7

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^{x}$, $y = (0.97)^{x}$, $y = (1.01)^{12x}$, $y = (1/2)^{x/10}$, and classify them as representing exponential growth or decay.

NO CCCs linked to F-IF.8

9. Compare properties of two functions each represented in a different way (algebraically,graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

NO CCCs linked to F-IF.9

Building Functions—F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

| NO CCCs linked to F-BF.1 |
|---|

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

| NO CCCs linked to F-BF.2 |
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Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f (kx), and f (x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

| NO CCCs linked to F-BF.3 |
|---|

4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2x3 or f(x) = (x+1)/(x-1) for ≠ 1.
    b. Verify by composition that one function is the inverse of another.
    c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. Produce an invertible function from a non-invertible function by restricting the domain.

| NO CCCs linked to F-BF.4 |
|---|

5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

| NO CCCs linked to F-BF.5 |
|---|

Linear, Quadratic, and Exponential Models—F-LE

Construct and compare linear, quadratic, and exponential models, and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

| CCCs linked to F-LE.1 |
|---|
| H.PRF.1c1 Select the appropriate graphical representation of a linear model based on real-world events. |
| H.PRF.1b1 In a linear situation using graphs or numbers, predict the change in rate based on a given change in one variable (e.g., If I have been adding sugar at a rate of 1T per cup of water. What happens to my rate if I switch to 2T of sugar for every cup of water?). |

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

| NO CCCs linked to F-LE.2 |
|---|

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

| CCCs linked to F-LE.3 |
|---|
| H.PRF.2c1 Make predictions based on a given model (for example, a weather model, data for athletes over years). |

4. For exponential models, express a logarithm the solution to $abc^x = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

| NO CCCs linked to F-LE.4 |
|---|

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

| NO CCCs linked to F-LE.5 |
|---|

Trigonometric Functions—F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

| NO CCCs linked to F-TF.1 |
|---|

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

| NO CCCs linked to F-TF.2 |
|---|

3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

| NO CCCs linked to F-TF.3 |
|---|

4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

| NO CCCs linked to F-TF.4 |
|---|

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

| NO CCCs linked to F-TF.5 |
|---|

6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

| NO CCCs linked to F-TF.6 |
|---|

7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.

| NO CCCs linked to F-TF.7 |
|---|

Prove and apply trigonometric identities

8. Prove the Pythagorean identify $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

| NO CCCs linked to F-TF.8 |
|---|

9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

| NO CCCs linked to F-TF.9 |
|---|

## Core Content Connectors by Common Core State Standards: Mathematics, Algebra

MSAA Instructional Resource Guide, Revised May 2024 from the NCSC content developed as part of the National Center and State Collaborative under a grant from the US Department of Education.

This view is designed to mirror the Common Core document while also including the CCCs linked to each CCSS.

Mathematics High School—Functions Overview
Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric functions

Interpreting Functions—F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

| NO CCCs linked to F-IF.1 |
|---|

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

| NO CCCs linked to F-IF.2 |
|---|

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f (0) = f (1) = 1, f (n + 1) = f (n) + f (n– 1) for n ≥ 1.

| NO CCCs linked to F-IF.3 |
|---|

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimum; symmetries; and behavior; and periodicity.

| NO CCCs linked to F-IF.4 |
|---|

5. Relate the domain of a function to its graph and where applicable to the quantitative relationships it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

| NO CCCs linked to F-IF.5 |
|---|

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

| NO CCCs linked to F-IF.6 |
|---|

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

No CCCs linked to F-IF.7

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^x$, $y = (0.97)^x$, $y = (1.01)12^x$, $y = (1.2)^x/10$, and classify them as representing exponential growth or decay.

| NO CCCs linked to F-IF.8 |
| --- |

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

| NO CCCs linked to F-IF.9 |
| --- |

Building Functions-F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a colling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, T(h(t)) is the temperature at the location of the weather balloon as a function of time.

| NO CCCs linked to F-BF.1 |
|---|

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

| NO CCCs linked to F-BF.2 |
|---|

Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustration and explanation of the effects of the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

| NO CCCs linked to F-BF.3 |
|---|

4. Find the inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2 x 3 or f(x) = (x + 1)/ (x – 1) for x ≠ 1.
    b. Verify by composition that one function from a graph or a table, given that the function has an inverse.
    c. Produce an invertible function from a non-invertible function by restricting the domain.

| NO CCCs linked to F-BF.4 |
|---|

5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

| NO CCCs linked to F-BF.5 |
|---|

Linear, Quadratic, and Exponential Models—F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

CCCs linked to F-LE.1
H.PRF.1c1 Select the appropriate graphical representation of a linear model based on real-world events.
H.PRF.1b1 In a linear situation using graphs or numbers, predict the change in rate based on a given change in one variable (e.g., If I have been adding sugar at a rate of 1T per cup of water. What happens to my rate if I switch to 2T of sugar for every cup of water?).

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

NO CCCs linked to F-LE.2

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

CCCs linked to F-LE.3
H.PRF.2c1 Make predictions based on a given model (for example, a weather model, data for athletes over years).

4. For exponential models, express a logarithm the solution to $abc^x = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

NO CCCs linked to F-LE.4

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear exponential function in terms of a context.

NO CCCs linked to F-LE.5

Trigonometric Functions—F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

| NO CCCs linked to F-TF.1 |
|---|

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

| NO CCCs linked to F-TF.2 |
|---|

3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$ and use the unit circles to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

| NO CCCs linked to F-TF.3 |
|---|

4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

| NO CCCs linked to F-TF.4 |
|---|

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

| NO CCCs linked to F-TF.5 |
|---|

6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

| NO CCCs linked to F-TF.6 |
|---|

7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.

| NO CCCs linked to F-TF.7 |
|---|

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

| NO CCCs linked to F-TF.8 |
|---|

9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems

| NO CCCs linked to F-TF.9 |
|---|

## Core Content Connectors by Common Core State Standards: Mathematics, Algebra

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Mathematics High School—Algebra Overview
Seeing Structure in Expressions

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Seeing structure in Expressions—A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P (1 + r) n as the product of P and a factor not depending on P.

| CCCs linked to A-SSE.1 |
|---|
| H.PRF.2a1 Translate an algebraic expression into a word problem. |

2. Use the structure of an expression to identify ways to rewrite it. for example, see x4 – y4 as (x2) – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2) (x2 + y2).

| CCCs linked to A-SSE.2 |
|---|
| H.NO.2c1 Simplify expressions that include exponents. |
| H.NO.2c2 Rewrite expressions that include rational exponents. |

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    b. Complete the square in a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the express 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

| CCCs linked to A-SSE.3 |
|---|
| H.NO.1a1 Represent quantities and expressions that use exponents. |
| H.PRF.2a2 Factor a quadratic expression. |
| H.PRF.2a3 Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression (x – a)(x – c), a and c correspond to the x-intercepts (if a and c are real). |

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. For example, calculate mortgage payments.

| CCCs linked to A-SSE.4 |
|---|
| H.PRF.2a4 Use the formula to solve real world problems such as calculating the height of a tree after n years given the initial height of the tree and the rate the tree grows each year. |

Arithmetic with Polynomials and Rational Expressions—A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

| CCCs linked to A-APR.1 |
| --- |
| H.NO.2a2 Understand the definition of a polynomial. |
| H.NO.2a3 Understand the concepts of combining like terms and closure. |
| H.NO.2a4 Add, subtract, and multiply polynomials and understand how closure applies under these operations. |

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

| CCCs linked to A-APR.2 |
| --- |
| H.NO.2a5 Understand and apply the Remainder Theorem. |

3. Identify zeroes of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

| CCCs linked to A-APR.3 |
| --- |
| H.NO.2a6 Find the zeros of a polynomial when the polynomial is factored. |

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples.

| CCCs linked to A-APR.4 |
| --- |
| H.NO.3a6 Prove polynomial identities by showing steps and proving reasons. |
| H.NO.3a7 Illustrate how polynomial identities are used to determine numerical relationships such as 2 2 2 2 25 = (20 + 5) = 20 + 2 x 20 x 5 + 5 |

5. Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle 1.

| NO CCCs linked to A-APR.5 |
| --- |

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or for the more complicated examples, a computer algebra system.

| CCCs linked to A-APR.6 |
| --- |
| H.PRF.2a5 Rewrite rational expressions, a(x)/b(x), in the form q(x) + r(x)/b(x) by using factoring, long division, or synthetic division. |

7. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

NO CCCs linked to A-APR.7

Creating Equations—A-CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

| CCCs linked to A-CED.1 |
| --- |
| H.PRF.2b1 Translate a real-world problem into a one variable linear equation. |

2. Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales.

| CCCs linked to A-CED.2 |
| --- |
| H.PRF.2b2 Solve equations with one or two variables using equations or graphs. |

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

| CCCs linked to A-CED.3 |
| --- |
| H.PRF.2a6 Write and use a system of equations and/or inequalities to solve a real-world problem. |

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's Law V = IR to highlight resistance R.

| CCCs linked to A-CED.4 |
| --- |
| H.PRF.1b2 Solve multi-variable formulas or literal equations, for a specific variable. |

Reasoning with Equations and Inequalities—A-REI

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

| CCCs linked to A-REI.1 |
|---|
| H.PRF.2b2 Solve equations with one or two variables using equations or graphs. |

2. Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

| CCCs linked to A-REI.2 |
|---|
| H.NO.2a1 Solve simple equations using rational numbers with one or more variables. |

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

| CCCs linked to A-REI.3 |
|---|
| H.PRF.2b2 Solve equations with one or two variables using equations or graphs. |
| H.ME.1b2 Solve a linear equation to find a missing attribute given the area, surface area, or volume and the other attribute. |

4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform and quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

| CCCs linked to A-REI.4 |
|---|
| H.PRF.2b3 Transform a quadratic equation written in standard form to an equation in vertex form (x – p) = q2 by completing the square. |
| H.PRF.2b4 Derive the quadratic formula by completing the square on the standard form of a quadratic equation. |
| H.PRF.2b5 Solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square. |

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

| CCCs linked to A-REI.5 |
|---|
| H.PRF.2b6 Solve systems of equations using the elimination method (sometimes called linear combinations). |
| H.PRF.2b7 Solve a system of equations by substitution (solving for one variable in the first equation and substitution it into the second equation). |

6. Solve systems of linear equations exactly (e.g., with graphs), focusing on pairs of linear equations in two variables.

| |
|---|
| CCCs linked to A-REI.6<br>H.PRF.2b8 Solve systems of equations using graphs. |

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x2 + y2 = 3.

| |
|---|
| CCCs linked to A-REI.7<br>H.PRF.2b9 Solve a system containing a linear equation and a quadratic equation in two variables graphically and symbolically. |

8. Represent a system of linear equations as a single matrix equation in a vector variable.

| |
|---|
| NO CCCs linked to A-REI.8 |

9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

| |
|---|
| NO CCCs linked to A-REI.9 |

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

| |
|---|
| CCCs linked to A-REI.10<br>H.PRF.2b10 Understand that all solutions to an equation in two variables are contained on the graph of that equation. |

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

| |
|---|
| CCCs linked to A-REI.11<br>H.PRF.2d1 Explain why the intersection of y = f(x) and y = g(x) is the solution of f(x) = g(x) for any combination of linear or exponential. Find the solution(s) by: Using technology to graph the equations and determine their point of intersection, using tables of values, or using successive approximations that become closer and closer to the actual value. |

12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes.

| |
|---|
| CCCs linked to A-REI.12<br>H.PRF.2b11 Graph the solutions to a linear inequality in two variables as a halfplane, excluding the boundary for nonexclusive inequalities.<br>H.PRF.2b12 Graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding halfplanes. |

**Core Content Connectors by Common Core State Standards: Mathematics, Number, and Quantity**

MSAA Instructional Resource Guide, Revised June 2024 from the NCSC content developed as part of the National Center and State Collaborative under a grant from the US Department of Education.

This view is designed to mirror the Common Core document whole also including the CCCs linked to each CCSS.

Mathematics High School—Number and Quantity Overview
The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers

Quantities

- Reason quantitatively and use units to solve problems

The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matric Quantities

- Represent and model with vector quantities
- Perform operations on vectors
- Perform operations on matrices and use matrices in applications

The Real Number System—N-RN

Extend the properties of exponents to rational exponents

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{(1/3)})^3 = 5^{(1/3)})^3$ to hold, so $5^{(1/3)})^3$ must equal 5.

| NO CCCs linked to N-RN.1 |
|---|

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

| CCCs linked to N-RN.2 |
|---|
| HS.NO.1a1 Simplify expressions that include exponents. |
| HS.NO.1a2 Explain the influence of an exponent on the location of a decimal point in each manner. |
| HS.NO.1a3 Convert a number expressed in scientific notation. |
| H.NO.2c2 Rewrite expressions that include rational exponents. |

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

| CCCs linked to N-RN.3 |
|---|
| H.NO.2b1 Explain the pattern for the sum or product for combinations of rational and irrational numbers. |

Quantities—N-Q

Reason quantitatively and use units to solve problems

1. Use units to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

| CCCs linked to N-Q.1 |
|---|
| H.ME.1a1 Determine the necessary unit(s) to use to solve real-world problems. |
| H.ME.1a2 Solve real-world problems involving units of measurement. |

2. Define appropriate quantities for the purpose of descriptive modeling.

| NO CCCs linked to N-Q.2 |
|---|

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

| NO CCCs linked to N-Q.3 |
|---|

The Complex Number System—N-CN

Perform arithmetic operations with complex numbers

1. Know there is a complex number i such that $i^2 = 1$, and every complex number has the form a + bi with a and b as real numbers.

NO CCCs linked to N-CN.1

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

NO CCCs linked to N-CN.2

3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

NO CCCs linked to N-CN.3

Represent complex numbers and their operations on the complex plane.

4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.

NO CCCs linked to N-CN.4

5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.

NO CCCs linked to N-CN.5

6. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

NO CCCs linked to N-CN.6

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

NO CCCs linked to N-CN.7

8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.

NO CCCs linked to N-CN.8

9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

NO CCCs linked to N-CN.9

Vector and Matric Quantities-N-VM

Represent and model with vector quantities

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).

| NO CCCs linked to N-VM.1 |
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2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

| NO CCCs linked to N-VM.2 |
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3. Solve problems involving velocity and other quantities that can be represented by vectors.

| NO CCCs linked to N-VM.3 |
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Perform operations on vectors

4. Add and subtract vectors.
   a. Add vectors end to end, component wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction v – w as v + (-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.

| NO CCCs linked to N-VM.4 |
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5. Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c (vx, vy) = cvx, cvy).
   b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

| NO CCCs linked to N-VM.5 |
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Perform operations on matrices and use matrices in applications.

6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

| NO CCCs linked to N-VM.6 |
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7. Multiple matrices by scalars to produce new matrices, e.g., as when all the payoff in a game is doubled.

| NO CCCs linked to N-VM.7 |
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8. Add, subtract, and multiply matrices of appropriate dimensions.

| NO CCCs linked to N-VM.8 |
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9. Understand that unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

| NO CCCs linked to N-VM.9 |
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10. Understand that the zero and identity matrices play a role in matrix addition and multiplication like the role of 0 and 1 in the real numbers. The determinant of a sequence matrix is nonzero if and only if the matric has a multiplicative inverse.

| CCCs linked to N-VM.10 |
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11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

| NO CCCs linked to N-VM.11 |
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12. Work with 2 x 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area.

| NO CCCs linked to N-VM.12 |
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