

Patterns, Relations, and Functions 1

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(K-4) Elementary School Learning Targets

E.PRF-1 Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change.

(5-8) Middle School Learning Targets

M.PRF-1 Describe and compare situations that involve change and use the information to draw conclusions:

- Model contextual situations using multiple representations.
- Calculate rates of change for real-world situations (constant).

(9-12) High School Learning Targets

H.PRF-1 Approximate, calculate, model, and interpret change:

- Use graphical and numerical data resulting from complex situations.
- Model complex real-world phenomena to make predictions and provide explanations.

Grade Differentiation

Elementary School Progress Indicators

Progress Indicator: E.PRF.1b exploring and describing how addition or subtraction changes a quantity

K.PRF.1b1 Use objects or pictures to respond appropriately to "add __" and "take away _"

Operations and Algebraic Thinking

K.OA Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.

K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings¹, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

(¹Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.))

K.PRF.1b2 Communicate answer after adding or taking away

Operations and Algebraic Thinking

K.OA Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.

K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings¹, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

(¹ Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.))

Progress Indicator: E.PRF.1c modeling problem solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols

K.PRF.1c1 Solve one step addition and subtraction word problems, and add and subtract within 10 using objects, drawings, pictures

Operations and Algebraic Thinking

K.OA Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.

K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Progress Indicator: E.PRF.1b exploring and describing how addition or subtraction changes a quantity

1.PRF.1b3 Using objects or pictures respond appropriately to "add _" and "take away _"

Operations and Algebraic Thinking

1.OA Represent and solve problems involving addition and subtraction.

1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Progress Indicator: E.PRF.1c modeling problem solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols

1.PRF.1c2 Solve one step addition and subtraction word problems where the change or result is unknown ($4 + _ = 7$) or ($4 + 3 = _$), within 20 using objects, drawings, pictures
Operations and Algebraic Thinking

1 OA Represent and solve problems involving addition and subtraction.

1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Progress Indicator: E.PRF.1c modeling problem solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols

2.PRF.1c3 Solve one or two step addition and subtraction problems, and add and subtract within 100, using objects, drawings, pictures
Operations and Algebraic Thinking

2 OA Represent and solve problems involving addition and subtraction.

2.OA.A.1 Use addition and subtraction within 100 to solve one and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

2.PRF.1c4 Use pictures, drawings or objects to represent the steps of a problem
Operations and Algebraic Thinking

2 OA Represent and solve problems involving addition and subtraction.

2.OA.A.1 Use addition and subtraction within 100 to solve one and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

2.PRF.1c5 Write or select an equation representing the problem and its solution
Operations and Algebraic Thinking

2 OA Represent and solve problems involving addition and subtraction.

2.OA.A.1 Use addition and subtraction within 100 to solve one and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Progress Indicator: E.PRF.1d describing and modeling how addition, subtraction, multiplication, or division changes a quantity, including with fractions

3.PRF.1d1 Use objects to model multiplication and division situations involving up to 5 groups with up to 5 objects in each group and interpret the results

Operations and Algebraic Thinking

3 OA Represent and solve problems involving multiplication and division.

3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as several shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which several shares or several groups can be expressed as $56 \div 8$.*

Progress Indicator: E.PRF.1e using representations (tables, graphs, equations) to show how values of one quantity are related to values of another and to draw conclusions

3.PRF.1e1 Describe the rule for a numerical pattern (e.g., increase by 2, 5 or 10)

Operations and Algebraic Thinking

3 OA Solve problems involving the four operations and identify and explain patterns in arithmetic.

3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

3.PRF.1e2 Select or name the 3 next terms in a numerical pattern where numbers increase by 2, 5 or 10

Operations and Algebraic Thinking

3 OA Solve problems involving the four operations and identify and explain patterns in arithmetic.

3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Progress Indicator: E.PRF.1f representing and explaining equivalence concretely, graphically, and symbolically (equations, rules)

3.PRF.1f 1 Determine the equivalence between number of minutes and the fraction of the hour (e.g., 30 minutes = $\frac{1}{2}$ hour)

Measurement and Data

3 MD Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.

3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.PRF.1f 2 Determine the equivalence between the number of minutes and the number of hours (e.g., 60 minutes = 1 hour)

Measurement and Data

3 MD Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.

3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Progress Indicator: E.PRF.1d describing and modeling how addition, subtraction, multiplication, or division changes a quantity, including with fractions

3.PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results

Operations and Algebraic Thinking

3 OA Represent and solve problems involving multiplication and division.

3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

Progress Indicator: E.PRF.1e using representations (tables, graphs, equations) to show how values of one quantity are related to values of another and to draw conclusions

4.PRF.1e3 Solve multiplicative comparisons with an unknown using up to 2-digit numbers with information presented in a graph or word problem (e.g., an orange hat cost \$3. A purple hat cost 2 times as much. How much does the purple hat cost? [$3 \times 2 = p$])

Operations and Algebraic Thinking

4 OA Use the four operations with whole numbers to solve problems.

3.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Progress Indicator: E.PRF.1f representing and explaining equivalence concretely, graphically, and symbolically (equations, rules)

4.PRF.1f3 Apply the distributive property to solve problems with models

Measurement and Data

3 MD Geometric measurement: understand concepts of area and relate to multiplication and to addition.

3.MD.C.7c Relate area to the operations of multiplication and addition.

c) Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

4.PRF.1f4 Solve a 2-digit by 1-digit multiplication problem using 2 different strategies
Number and Operations in Base Ten

4 NBT Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Progress Indicator: M.PRF.1a describing how multiplication or division changes a quantity, including with fractions or decimals

5.PRF.1a1 Determine whether the product will increase, or decrease based on the multiplier

Number and Operations – Fractions

5 NF Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3.NF.B.5 Interpret multiplication as scaling (resizing), by:

a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Progress Indicator: M.PRF.1b distinguishing linear from nonlinear relationships as represented in graphical and tabular representations

5.PRF.1b1 Given 2 patterns involving the same context (e.g., collecting marbles) determine the 1st 5 terms and compare the values

Day	Joe	Kim
1	2	4
2	4	8
3	6	12
4	8	16

Operations and Algebraic Thinking

5 OA Analyze patterns and relationships.

5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

5.PRF.1b2 When given a line graph representing two arithmetic patterns, identify the relationship between the two

Operations and Algebraic Thinking

5 OA Analyze patterns and relationships.

5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Middle School Progress Indicators

Progress Indicator: M.PRF.1a describing how multiplication or division changes a quantity, including with fractions or decimals

5.PRF.1a2 Determine whether the quotient will increase or decrease based on the divisor
Number and Operations – Fractions

5 NF Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.5 Interpret multiplication as scaling (resizing), by:

a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Progress Indicator: M.PRF.1c comparing two rates and evaluating them for a given situation (e.g., best value)

6.PRF.1c1 Describe the ratio relationship between two quantities for a given situation
Ratios and Proportional Relationships

5 RP Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.PRF.1c2 Represent proportional relationships on a line graph

Ratios and Proportional Relationships

6 RP Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."*

Progress Indicator: M.PRF.1d using symbolic equations to summarize how the quantity of something changes

6.PRF.1d1 Solve real world single step linear equations

Expressions and Equations

6 EE Reason about and solve one-variable equations and inequalities.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Progress Indicator: M.PRF.1e representing and computing unit rates associated with ratios of lengths, areas, and other quantities measured in like or different units

6.PRF.1e1 Determine unit rates associated with ratios of lengths, areas, and other quantities measured in like units

Ratios and Proportional Relationships

7 RP Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1}{2} / \frac{1}{4}$ miles per hour, equivalently 2 miles per hour.*

7.PRF.1e2 Represent proportional relationships on a line graph

Ratios and Proportional Relationships

7 RP Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Progress Indicator: M.PRF.1f identifying essential quantitative relationship(s) in a situation and using symbolic expressions to represent it and draw reasonable conclusions from it

7.PRF.1f1 Use proportional relationships to solve multistep percent problems in real world situations

Ratios and Proportional Relationships

7 RP Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Progress Indicator: M.PRF.1g modeling, solving, and explaining contextualized problems using various representations such as graphs, tables, functions, and equations

7.PRF.1g1 Solve real world multi step problems using whole numbers

Expressions and Equations

7 EE Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.PRF.1g2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities

Expressions and Equations

7 EE Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a) Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b) Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make and describe the solutions.*

Progress Indicator: M.PRF.1e representing and computing unit rates associated with ratios of lengths, areas, and other quantities measured in like or different units

8.PRF.1e2 Represent proportional relationships on a line graph

Expressions and Equations

7 EE Understand the connections between proportional relationships, lines, and linear equations.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Progress Indicator: M.PRF.1f identifying essential quantitative relationships in a situation and using symbolic expressions to represent it and draw reasonable conclusions from it
8.PRF.1f 2 Describe or select the relationship between the two quantities given a line graph of a situation

Functions

8 F Use functions to model relationships between quantities.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Progress Indicator: M.PRF.1g modeling, solving, and explaining contextualized problems using various representations such as graphs, tables, functions, and equations

8.PRF.1g3 Solve linear equations with 1 variable

Expressions and Equations

8 EE Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different).

b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.PRF.1g4 Solve systems of two linear equations in two variables and graph the results.

Expressions and Equations

8 EE Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b) Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

8.PR.G.1g5 Solve real world and mathematical problems leading to two linear equations in two variables.

Expressions and Equations

8 EE Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b) Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

High School Progress Indicators

Progress Indicator: H.PRF.1a approximating, calculating, and interpreting rates of change using graphical and numerical data

H.PRF.1a1 Interpret the rate of change using graphical representations

Interpreting Categorical and Quantitative Data

S-ID Interpret linear models.

HSS-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of data.

Progress Indicator: H.PRF.1b exploring how the rate of change of something depends on how much there is of something else (as the rate of change of speed is proportional to the amount of force acting)

H.PRF.1b1 In a linear situation using graphs or numbers, predicts the change in rate based on a given change in one variable (e.g., If I have been adding sugar at a rate of 1T per cup of water. What happens to my rate if I switch to 2T of sugar for every cup of water?)

Linear, Quadratic, and Exponential Models

F-LE Construct and compare linear, quadratic, and exponential models and solve problems.

HSF-LE.A.1b Distinguish between situations that can be modeled with linear functions and with exponential functions.

b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to one another.

Progress Indicator: H.PRF.1c creating mathematical models, using rules and relationships to describe and predict objects and events in the real world

H.PRF.1c1 Select the appropriate graphical representation of a linear model based on real world events

HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.